An Efficient Optimization Approach to Real-Time Coordinated and Integrated Freeway Traffic Control

Amir Hosein Ghods, *Student Member, IEEE*, Liping Fu, and Ashkan Rahimi-Kian, *Senior Member, IEEE*

***Abstract*—This paper tackles the problem of real-time optimal control of traffic flow in a freeway network deployed with coordinated and integrated traffic controllers. One promising approach to this problem is casting the underlying dynamic control problem in a model predictive framework. The challenge is that the resulting optimization problem is computationally intractable for online applications in a network with a large number of controllers. In this paper, a game-theoretic approach with distributed controllers is proposed to address the foregoing issue. The efficiency of the proposed method is tested for a coordinated ramp metering and variable-speed limit control applied to a stretch of freeway network. The parallel nature of the optimization algorithm makes it suitable for solving large-scale problems with high accuracy. The speed and accuracy of the proposed solution approach are examined and compared with that of the conventional optimization method in a case study to demonstrate its superior performance.**

***Index Terms*—Distributed controllers, game theory, model predictive control (MPC), parallel optimization, ramp metering, speed limit control.**

I. Introduction

Several methods have been developed to improve the performance of freeway networks. Among them, control strategies such as ramp metering, speed limits, and route recommendation are recognized as the most effective ways to relieve the freeway traffic congestion. Furthermore, the latest advances in computers and communication technologies have made it feasible to implement network-wide multiple traffic control systems, as opposed to single local control schemes. Intuitively, for a given traffic network, more controllers could result in better performance. Nevertheless, for a network-wide implementation, the amount of data and the computational complexity of the underlying control algorithms quickly increase as the number of control measures increases. Therefore, in general, there exists a tradeoff between the quality of the control method and the amount of information and computational resources required to achieve that quality.

Traffic control strategies can generally be classified into three categories. The first category consists of offline or open-loop strategies, in which only historical data are used in deriving the controls. A good example of open-loop strategies is the fixed-time ramp metering [1], in which the control strategies are predetermined for a particular time of day by solving a linear programming problem based on historical demand. A more sophisticated strategy in this category is the nonlinear optimal ramp-metering method [2], which attempts to minimize an objective function for the whole network. One of the major drawbacks of this control strategy is its high sensitivity to inaccuracies in the predicted traffic demands, traffic patterns, and incidents.

The second category contains the reactive or close-loop methods, which derive the control decisions based on real-time data from traffic sensors such as inductive loop detectors. Generally, this type of controller aims at keeping the freeway conditions as close to a prespecified target state as possible. Reactive ramp metering algorithms such as demand–capacity strategy [3] and ALINEA [4] are popular in this category. These controls do not incorporate any systematic optimization procedure to directly minimize the objective function and are mostly heuristic in nature, and their performance depends on the appropriate selection of the control parameters. Reference [5] provides a comprehensive review of the various ramp-metering methods in these two categories.

The third category includes control strategies commonly called proactive or predictive control methods that make use of both offline and online information to predict the future state of the underlying network and then control the system accordingly. The goal of these strategies is to find the optimal control over a given horizon based on a predefined objective function. It operates in a feedback adaptive fashion by which it takes new observed states and disturbances into account through a prediction model. These control methods are commonly referred to as receding horizon control or model predictive control (MPC). The MPC has been applied in ramp metering [6], variable speed limits [7], combined ramp metering and variable speed limit control [8], and combined dynamic route guidance and ramp-metering control [9].

Despite the obvious advantages of online strategies with optimization frameworks, such as the MPC, they have the drawback that their computational complexity quickly increases by the number of control inputs. This is particularly problematic for

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A. H. Ghods was with the Control and Intelligent Processing Center of Excellence, School of Electrical and Computer Engineering, College of Engineering, University of Tehran, Tehran 14395-515, Iran. He is now with the Transportation Systems Research Group, Department of Civil and Environmental Engineering, University of Waterloo, Waterloo, ON N2L 3G1, Canada (e-mail: ahghods@uwaterloo.ca).

L. Fu is with the Transportation Systems Research Group, Department of Civil and Environmental Engineering, University of Waterloo, Waterloo, ON N2L 3G1, Canada (e-mail: lfu@uwaterloo.ca).

A. Rahimi-Kian is with Control and Intelligent Processing Center of Excellence, School of Electrical and Computer Engineering, College of Engineering, University of Tehran, Tehran 14395-515, Iran (e-mail: arkian@ut.ac.ir).

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traffic-control systems where a closed-form optimal control signal may not explicitly be derived, and for each control interval, an online nonlinear programming technique must be implemented. For instance, Di Febbraro *et al.* [10] proposed to apply artificial neural networks as an offline control for optimal freeway traffic control instead of using online optimization for their receding horizon approach because they found that the dynamics of the system change faster than the speed of the computing system. In another case [8], it was suggested that a hierarchical control scheme be tested that was decomposing the large traffic network into small subnetworks with minimum interaction and then solving each problem locally. In [11], a hierarchical control structure is proposed to the coordinated ramp-metering problem arising in the Amesterdam ring road. The problem was formulated with a nonlinear macroscopic traffic model. The solution method proposed in that work was claimed to be fast enough for real-time implementation; however, it is unknown whether this solution approach could be extended to solve problems with more sophisticated controllers (e.g., speed limits) and input/state constraints. Despite the computational challenges, the potential of online control strategies like the MPC is very promising, and the remaining challenge is to develop a solution method that can feasibly be implemented in a real-world setting.

In this paper, we consider the problem of applying the MPC control framework to the congestion control problem of a freeway network equipped with ramp metering and variable speed limits. A solution algorithm from game theory is proposed to find the optimal solutions for the optimization part of the MPC, which has the potential to make the real-time congestion control computationally tractable even for large traffic networks. A macroscopic traffic flow model is used as the prediction model of the real traffic system. This paper is organized as follows: In Section II, the problem description is presented. In Section III, the basics of the MPC are introduced. In Section IV, the traffic flow model (prediction model) is introduced. In Section V, the problem formulation is proposed. The game-theoretic approach is explained in Section VI. The proposed method is applied to a benchmark problem in Section VII. Finally, conclusions are stated in Section VIII.

II. Integrated and Coordinated Control Problem

We consider the problem of finding the best control settings for a group of controllers in a traffic network consisting of a set of ramp meters and variable speed limit signs. The control objective is to minimize the system-wide total time spent (TTS) by all vehicles in the freeway network. Ramp metering is the most widely used freeway traffic-control method around the world. However, this method will lose its effectiveness as the congestion level increases. Changing the speed limit through variable speed limit signs could partially address this issue and improve the effectiveness of the ramp-metering system, as shown in [8]. The speed limiters located just before the bottleneck on-ramp can help reduce the outflow of controlled segments so that there will be some space left to accommodate the traffic from the on-ramp. This way, the traffic flow in the on-ramp area could be kept near the capacity, and the duration of breakdowns could be reduced. Therefore, a combination of ramp metering and variable speed limit control has the potential to achieve better performance than when they are implanted separately.

Coordination among different controllers that work together is an essential task. For instance, a controller at one spot of a freeway network may mitigate a local congestion problem but may induce congestion at another location on the freeway. Besides using the global data, the prediction of network evolution could be valuable since the effect of control can be seen after a time delay.

As the number of ramp meters and speed control limits increases, the size of the solution vector grows rapidly. For example, to find an optimal solution for N controllers including ramp meters and speed limiters using the MPC approach, (which will be explained in the next section), every controller must find C optimal values at each control time step. Therefore, the solution to the optimal control problem is an $N \times C$ variable matrix. If the problem is formulated as an integer-programming problem with S discrete permissible values for each $N \times C$ variable matrix, then $S^{N \times C}$ values have to be enumerated and evaluated to find the global optimal solution. Although the problem could also be formulated as a continuous nonlinear programming problem, the resulting problem is likely to be nonconvex in nature in that finding the global optimum solution would require an exhaustive search of the whole solution space.

III. Model Predictive Control

The MPC is an advanced control framework that was originally developed for industrial process control (see [12] and [13]). The MPC is a distinguished control model in terms of its capability to deal with various system constraints in an optimization framework. The core idea of the MPC is its use of a dynamic model to predict the future behavior of the system at each optimization step. The goal is to find the desired control inputs such that a predefined objective function is minimized or maximized. In this paper, we have utilized MPC as an online method to optimally control coordination of speed limits and ramp metering with the objective of minimizing the TTS with system states being predicted by a macroscopic freeway model. The following section provides a brief description of the MPC framework introduced in [14].

We consider a control system with N controllers over a specific time horizon. The time horizon is divided into P large control intervals, each subdivided into M small intervals (called system simulation steps). It is assumed that over each control interval, the control variables are kept the same, whereas the system state changes by the simulation step. Let k_c be the index for large intervals ($k_c = 1, 2, \ldots, P$) and k for all the subintervals ($k = 1, 2 \ldots, MP$). The transition of the system state can be expressed as follows:

$$\boldsymbol{x}(k+1) = f\left(\boldsymbol{x}(k), \boldsymbol{u}(k), \boldsymbol{d}(k)\right)$$

where $\boldsymbol{x}(k)$, $\boldsymbol{u}(k)$, and $\boldsymbol{d}(k)$ are vectors representing the system state, the control input, and the disturbance at time k. At each control step k_c, a new optimization is performed to compute the optimal control decisions, e.g.,

$$\boldsymbol{u}(k_c) = \begin{bmatrix} u_1(k_c) & u_1(k_c+1) & \cdots & u_1(k_c+P-1) \\ & & \vdots & \\ u_N(k_c) & u_N(k_c+1) & \cdots & u_N(k_c+P-1) \end{bmatrix}$$

for the time period of $[1.2, \ldots, P]$, in which P is the prediction horizon.

To reduce the computational complexity, a control horizon $C (C < P)$ is usually defined to represent the time horizon over which the control signal is considered to be fixed, i.e.,

$$u(k_c) = u(C-1) \; for \; k_c > C.$$

Therefore, for N controllers, the $N \times C$ vector of optimal controls would be

$$\boldsymbol{u}^*(k_c) = \begin{bmatrix} u_1^*(k_c) & u_1^*(k_c+1) & \cdots & u_1^*(k_c+C-1) \\ & & \vdots & \\ u_N^*(k_c) & u_N^*(k_c+1) & \cdots & u_N^*(k_c+C-1) \end{bmatrix}.$$

Only the first optimal control signal $\boldsymbol{u}_i^*(k_c)$, $i = 1, 2, \ldots, N$ (first column) is applied to the real system, and after shifting the prediction and control horizon one step forward with the current observed states of the real system to the model, the process is repeated. This feedback is necessary to correct any prediction errors and system disturbances that may deviate from model prediction. Since we have to work with a nonlinear system (traffic model), in each control time step k_c, a nonlinear programming has to be solved to find the $N \times C$ optimal solutions before reaching the next control time step $(k_c + 1)$.

It should be pointed out that the control parameters P and C need to be selected appropriately. Choosing a large prediction and control horizon will increase the computational demands due to the increased number of optimization variables. On the other hand, using a short prediction and control horizon may turn the control strategy into a reactive model and thus degrade its effectiveness.

In the following sections, we introduce how the system state equations are modeled using a dynamic traffic flow model and how the MPC can be cast into a game-theoretical framework and solved efficiently.

IV. Traffic-Flow Model

The traffic-flow model adopted here is the destination independent METANET model (see [2] for more details) together with the extended model for speed limits presented in [8].

The METANET is a macroscopic traffic model that is discrete in both space and time. The model represents the network by a directed graph with a set of links corresponding to freeway

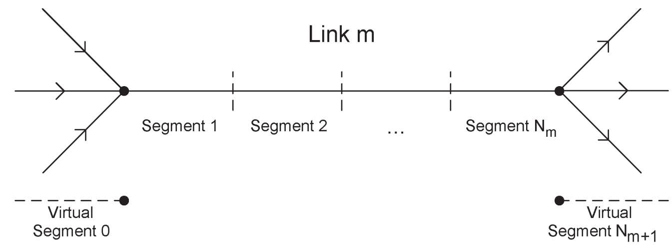

Fig. 1. METANET model. Link and node configuration.

stretches and a set of nodes, as illustrated in Fig. 1. Each link has uniform characteristics i.e., no on-ramp or off-ramp and no major changes in geometry. The nodes of the graph are placed between links, where the major change in road geometry occurs, such as on-ramps and off-ramps. A freeway link (m) is divided into (N_m) segments (indexed by i) of length $(l_{m,i})$ and by the number of lanes (n_m). Each segment (i) of link (m) at time instant $t = kT$, where T is the time step used for simulation, and $k = 0, \ldots, K$, is macroscopically characterized by its *traffic density* $\rho_{m,i}(k)$ (in vehicles per lane per kilometer), *mean speed* $v_{m,i}(k)$ (in kilometers per hour), and *traffic volume* $q_{m,i}(k)$ (in vehicles per hour). Table I describes the notations related to the METANET model.

The traffic stream models that capture the evolution of traffic on each segment at each time step are shown in (1)–(8) (see Table II). The node equations that represent the relation between connected links are given in (9)–(12) (see Table III), which show how the entering traffic flow to a node is distributed among the emanating links.

Using the aforementioned equations, the nonlinear traffic dynamics can be expressed as follows:

$$\boldsymbol{x}(k+1) = f\left(\boldsymbol{x}(k), \boldsymbol{u}(k), \boldsymbol{d}(k)\right) \tag{13}$$

where $\boldsymbol{x}(k)$ is the state vector of the system, that is, flow rate $(q_{m,i}(k))$, speed $(v_{m,i}(k))$, density $(\rho_{m,i}(k))$, and queue length of origins $w_o(k)$; $\boldsymbol{u}(k)$ is the vector of control inputs, including the ramp metering rates and the speed limits; and $\boldsymbol{d}(k)$ is the disturbance vector at simulation step k.

Based on $\boldsymbol{x}(k)$, $\boldsymbol{u}(k)$, and $\boldsymbol{d}(k)$, the future evolution of the traffic system $[\widehat{\boldsymbol{x}}(k+1), \ldots, \widehat{\boldsymbol{x}}(k+MP-1)]$ can be predicted by the METANET model.

V. Problem Formulation

With the definitions and system state equations introduced in the previous section, we can now present the formulation of the MPC optimization problem. The optimal control problem includes the following two sets of decision variables:

1) $v_i(j)$: variable speed limits for $j \in [k, \ldots, k+C-1]$ and $i \in I_{\text{speed}}$, where I_{speed} is the set of speed limits that are presented in the freeway network;
2) $r_o(j)$: ramp-metering rates for $j \in [k, \ldots, k+C-1]$ and $o \in O_{\text{ramp}}$, where O_{ramp} is the set of controlled on-ramps where ramp metering is presented.

TABLE I
NOTATIONS USED IN THE METANET MODEL

m, μ	Link index
i	Segment index
T	Simulation step size
k	Time step counter
$\rho_{m,i}(k)$	Density of segment i of freeway link m (veh/km/lane)
$\upsilon_{m,i}(k)$	Speed of segment i of freeway link m (km/h)
$q_{m,i}(k)$	Flow of segment i of freeway link m (veh/h)
N_m	Number of segments in link m
n_m	Number of lanes in link m
$l_{m,i}$	Length of segment i in link m (km)
τ	Time constant of the speed relaxation term (h)
κ	Speed anticipation term parameter (veh/km/lane)
υ	Speed anticipation term parameter (km^2/h)
α_m	Parameter of the fundamental diagram
$\rho_{crit,m}$	Critical density of link m (veh/km/lane)
$V(\rho_{m,i}(k))$	Speed of segment i of link m on a homogeneous freeway as a function of the density $\rho_{m,i}(k)$
$\rho_{max,m}$	Maximum density (veh/km/lane) of link m
$v_{free,m}$	Free-flow speed of link m (km/h)
$w_o(k)$	Length of the queue on on-ramp o at the time step k (veh)
$q_o(k)$	Flow that enters into the freeway at time sep k (veh/h)
$d_o(k)$	Traffic demand at origin o at time step k (veh/h)
$r_o(k)$	Ramp metering rate of on-ramp o at time step k
Q_o	On-ramp capacity (veh/h)
δ	Speed drop term parameter caused by merging at an on-ramp
n	Node index
Q_n	Total flow that enters freeway node n (veh/h)
I_n	Set of link indexes that enter node n
O_n	Set of link indexes that leave node n
β_n^m	Fraction of the traffic that leaves node n via link m
$v_{control,m,i}$	Speed limit applied in segment i of link m (km/h)
α	Parameter expressing the disobedience of drivers with the displayed speed limits

The objective function used in this paper is the TTS spent by all vehicles, as defined in

$$\begin{aligned}\text{TTS} &= J(v,r)\\ &= T\sum_{j=k}^{k+P-1}\left\{\sum_{m,i}\rho_{m,i}(j)l_{m,j}n_m+\sum_o w_o(j)\right\}\\ &\quad+\sum_{j=k}^{k+P-1}\left\{\alpha_{\text{ramp}}\sum_{o\in O_{\text{ramp}}}(r_o(j)-r_o(j-1))^2\right.\\ &\qquad\left.+\,\alpha_{\text{speed}}\sum_{i\in I_{\text{speed}}}\left(\frac{v_i(j)-v_i(j-1)}{v_{\text{free}}}\right)^2\right\}\\ &\quad+\,\alpha_{\text{queue}}\sum_{o\in O_{\text{ramp}}}\left(\max(w_o-w_{\max})\right)^2. \end{aligned} \tag{14}$$

The first two terms in (14) correspond to the main stream and the origins' queues, respectively. The second and third terms, which are weighted by nonnegative weighting factors, enable the control strategy to penalize abrupt changes in the ramp metering and speed-limit-control decisions, and the last term with a nonnegative weighting factor penalizes queue lengths larger than the on-ramp capacity for keeping the queue lengths within the permissible limit of the on-ramps.

The MPC optimization problem can therefore be formulated as follows in an abbreviated form:

$$\begin{aligned}\min \quad & \{J(\boldsymbol{v},\boldsymbol{r}) : \boldsymbol{v}\in\boldsymbol{V}, \boldsymbol{r}\in\boldsymbol{R}\}\\ \text{s.t.} \quad & \text{Equations (1)–(12)}\end{aligned} \tag{15}$$

where for N_1 speed limits and N_2 ramp meters, $\boldsymbol{v}(N_1\times C)$ and $\boldsymbol{r}(N_2\times C)$ are decision variables, respectively, $(N_1+N_2=N)$, and $\boldsymbol{V}\times\boldsymbol{R}$ is the feasible search space. We

TABLE II
LINK EQUATIONS AND DESCRIPTIONS

$q_{m,i}(k)=\rho_{m,i}(k)v_{m,i}(k)n_m$	(1)	Flow-Density-Speed equation
$\rho_{m,i}(k+1)=\rho_{m,i}(k)+\frac{T}{l_{m,i}n_m}[q_{m,i-1}(k)-q_{m,i}(k)]$	(2)	Conservation of vehicles
$v_{m,i}(k+1)=v_{m,i}(k)+\underbrace{\frac{T}{\tau_m}\{V[\rho_{m,i}(k)]-v_{m,i}(k)\}}_{Relaxation\,Term}+\underbrace{\frac{T}{l_{m,i}}v_{m,i}(k)[v_{m,i-1}(k)-v_{m,i}(k)]}_{Convection\,Term}-\underbrace{\frac{\vartheta_m T}{\tau_m.l_{m,i}}\frac{\rho_{m,i+1}(k)-\rho_{m,i}(k)}{\rho_{m,i}(k)+\kappa_m}}_{Anticipation\,Term}$	(3)	Speed dynamic Relaxation Term: drivers try to achieve desired speed $V(\rho)$. Convection Term: Speed decrease or increase caused by inflow of vehicles. Anticipation Term: the speed decrease (increase) as drivers experience the density increase (decrease) in downstream.
$V[\rho_{m,i}(k)]=v_{free,m}\exp\left(-\frac{1}{a_m}\left(\frac{\rho_{m,i}(k)}{\rho_{crit,m}}\right)^{a_m}\right)$	(4)	Speed-Density relation (fundamental diagram)
$w_o(k+1)=w_o(k)+T(d_o(k)-q_o(k))$	(5)	Origins' queueing model
$q_o(k)=\min\left[\begin{array}{l} d_o(k)+\frac{w_o(k)}{T}, Q_o.r_o(k), \\ Q_o\frac{\rho_{\max,m}-\rho_{m,1}(k)}{\rho_{\max,m}-\rho_{crit,m}}\end{array}\right]$	(6)	Ramp outflow equation The outflow depends on the traffic condition in the main-stream and also on the metering rate, $r_o(k)\in[0,1]$
$V(\rho_{m,i}(k))=\min\left\{\begin{array}{l} v_{free,m}\exp\left(-\frac{1}{a_m}\left(\frac{\rho_{m,i}(k)}{\rho_{crit,m}}\right)^{a_m}\right), \\ (1+\alpha)v_{control,m,i}(k)\end{array}\right\}$	(7)	Speed limit model The desired speed is the minimum of the speed determined by (4) and the speed limit, which is displayed on the variable message sign (VMS).
$-\frac{\delta T q_o(k)v_{m,1}}{l_{m,i}n_m(\rho_{m,1}(k)+\kappa)}$	(8)	Speed drop caused by merging phenomena. If there is an on-ramp then the term must be added to (3)

TABLE III
NODE EQUATIONS AND DESCRIPTIONS

$Q_n(k)=\sum_{\mu\in I_n}q_{\mu,N_\mu}(k)$	(9)	Total traffic flow enter node n
$q_{m,0}(k)=\beta_n^m(k).Q_n(k)$	(10)	Traffic flow that leaves node n via link m
$\rho_{m,N_{m+1}}(k)=\frac{\sum_{\mu\in O_n}\rho_{\mu,1}^2(k)}{\sum_{\mu\in O_n}\rho_{\mu,1}(k)}$	(11)	Virtual downstream density, when node n has more than one leaving link
$v_{m,0}(k)=\frac{\sum_{\mu\in I_n}v_{\mu,N_\mu}(k).q_{\mu,N_\mu}(k)}{\sum_{\mu\in I_n}q_{\mu,N_\mu}(k)}$	(12)	Virtual upstream speed, when node n has more than one entering link

call the whole decision variable vector $\boldsymbol{u}(N\times C)$, which is as follows:

$$\boldsymbol{u}(k_c)=\begin{bmatrix} v_1(k_c) & v_1(k_c+1) & \cdots & v_1(k_c+C-1) \\ & & \vdots & \\ v_{N_1}(k_c) & v_{N_1}(k_c+1) & \cdots & v_{N_1}(k_c+C-1) \\ r_1(k_c) & r_1(k_c+1) & \cdots & r_1(k_c+C-1) \\ & & \vdots & \\ r_{N_2}(k_c) & r_{N_2}(k_c+1) & \cdots & r_{N_2}(k_c+C-1) \end{bmatrix}.$$

Because of the nonlinearity of the traffic system states (1)–(12) and the objective function, this problem is a nonlinear programming with $N\times C$ decision variables. The problem is commonly solved using sequential quadratic programming (SQP) algorithm [8]. However, the SQP algorithm is viable only for small problems, and its optimality is not guaranteed. Therefore, to find a sufficiently good solution in a reasonable time for this problem, we apply the game theory that has successfully been applied to solve large-size optimization problems in other fields.

VI. GAME-THEORETIC APPROACH

The game theory was first introduced in the economy to find the market equilibrium when multiple firms compete with each other to sell or buy some goods. Game theory studies how rational decision makers (players) choose their strategies from the sets of decisions that depend on the strategies of other players. In other words, each player has a payoff function that is affected by the strategy of the player itself and the strategies of other players. There are two types of strategies defined in game theory: 1) If a player has a dominant strategy or knows what his/her opponent will do in the next step, then he/she could take a strategy with probability 1, which is called *pure strategy*. 2) However, in incomplete information games where players do not have dominant strategies or are not sure about the next step decisions of their rivals, they may assign different probabilities to their own and their rivals' decision sets, and their strategy vectors are called *mixed strategies* (for more details regarding game theory and applications, see [15] and [16]).

The basic idea of using game theory in this paper for freeway optimal traffic control is to decompose the whole optimization problem into a number of suboptimization problems with smaller dimensions and to solve them individually but in a coordinated way. This is similar to turning the optimization problem into a sequential and coordinated game that is played by a number of players with identical payoffs. In our case, each of the N controllers in the traffic network is considered as a player in a game, and the TTS of all vehicles in the network is considered the objective function of all the players. Therefore, the optimal coordination of the ramp metering and variable speed limits is presented as a game of identical interests.

Since the players (traffic controllers) decide simultaneously and try to chose their best strategies in response to the predicted strategies of their rivals (other network controllers), the solution vector of such game represents a state called *Nash equilibrium*, in which the players cannot improve their payoffs by changing their strategies unilaterally. The Nash equilibrium solution can be found through a well-known algorithm called fictitious play (FP) [17]. The FP is an interactive process in which the players find their best strategies by predicting the rivals' strategies based on the probability distributions of their past decisions. In general, the FP is not guaranteed to converge to the Nash equilibrium; however, it does converge to the Nash equilibrium in games of identical interest or common objective (in our case TTS) [18]. Virtually, the optimization problems may be viewed as a game of identical objectives in which the Nash solution has some optimality properties; as a result, the FP has recently become increasingly popular as an optimization tool.

The classical form of FP is computationally extensive in practice. Reference [19] proposed a modified form of it called sample FP (SFP) that is similar to the original FP with a difference that the best strategies are computed against a random sample from the history of the past decisions of the rivals instead of the predicted decisions based on their probability distributions. The SFP algorithm is useful to solve the problem of form (15), particularly when the objective function is evaluated through a black-box module requiring significant computational efforts for each function evaluation similar to our case (see [19] for more details). In the SFP method, each player finds its best strategy by assuming that other players

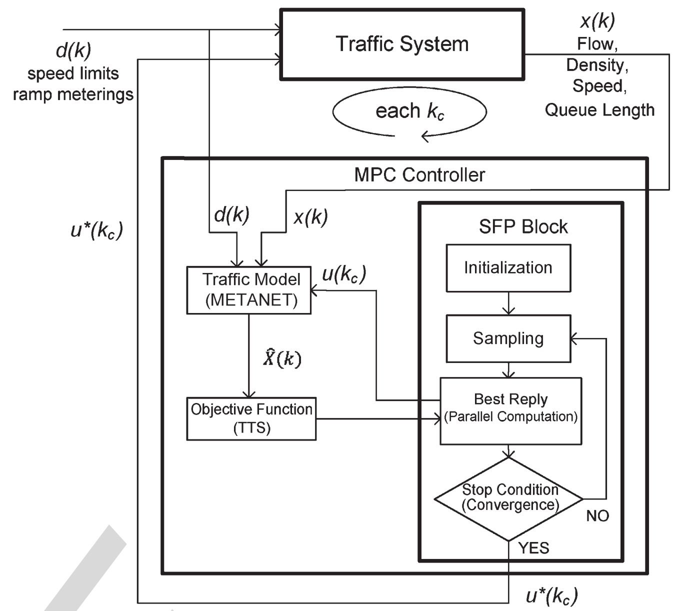

Fig. 2. Schematic diagram of MPC with SFP optimization method.

play known strategies drawn randomly from the history of their past plays. Therefore, players learn other players' strategies iteratively. The convergence of the SFP with the increasing number of iterations has also been proven in [19]. The SFP algorithm has been applied for solving the dynamic traffic-assignment problem [20], the communication protocol design [21], and the signalized intersection problem [22].

The SFP algorithm has the following steps, as reported in [22]:

1) Initialization: A set of initial strategies is randomly chosen for each player and stored in the history.
2) Sampling: A strategy arbitrarily drawn from the history of plays for each player with equal probability.
3) Best reply: Each player computes his/her best reply or strategy, assuming that other players play the strategies drawn in the previous step.
4) Store: The best replies obtained in Step 3 are stored in the history of plays.
5) Stop Condition: Check whether the stopping criterion is met (for example, if the solution vector has reached the steady-state Nash equilibrium); if not, then go to Step 2.

The most important feature of the SFP algorithm is that the best-reply computation can be done in parallel for all players simultaneously. This makes the algorithm feasible for parallel implementation, that is, the N, C-dimensional optimization problems can be solved in parallel. It is also possible to decompose the problem into much smaller subproblems by assuming the C control signal of each controller as an individual player. Accordingly, we would have $N \times C$ players, each with a 1-D optimization problem. We omitted this configuration because in this scheme the divergence time associated with $N \times C$ players might have become problematic as the number of controlled inputs would increase. Furthermore, the C-dimensional problem is small enough for our optimization algorithm, and

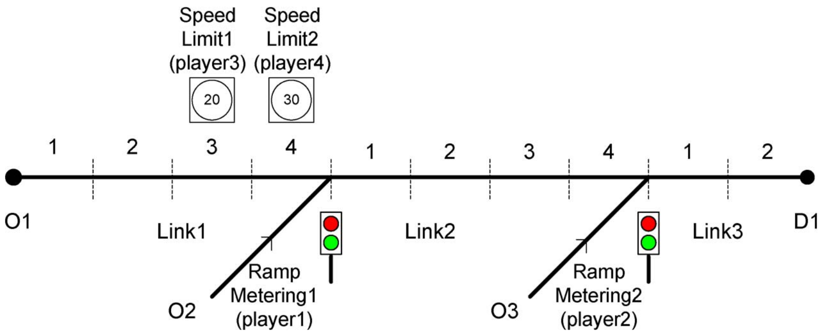

Fig. 3. Benchmark network with two on-ramp metering and two speed limits. Each controller has been considered as a player.

the parameter C does not vary as the number of controller increases.

The SFP algorithm of coordinated ramp metering and variable speed limits in the MPC framework can be presented as follows (see Fig. 2 for the schematic description):

1) Initialization: A set of initial values is randomly chosen for each of the ramp meters and speed limits for a given control horizon (C). $(\boldsymbol{u}_i^{\text{initial}}(1 \times C)$ for $i = 1, \ldots, N)$.
2) Sampling: The control values are arbitrarily drawn from the history of previously stored values for each controller with equal probability (equal to initial values for the first step). $(\boldsymbol{u}_i^{\text{history}}(1 \times C)$ for $i = 1, \ldots, N)$.
3) Optimization: Each controller finds its optimal values by minimizing the objective function of (14) over the prediction horizon, assuming that all the other controllers have taken constant values (drawn from Step 2). The METANET model is utilized as the prediction model and the SQP algorithm as a numerical optimization algorithm to find the optimal controls. $\boldsymbol{u}_i^*(1 \times C)$ for $i = 1, \ldots, N$.
4) Store: The new optimal values obtained in Step 3 are stored in the history of the players' decisions.
5) Stop Condition: Checks whether the convergence of the fitness function for each controller has occurred (i.e., if the steady-state Nash equilibrium has been reached). If yes, then stop and repeat this algorithm for the next iteration $(k+1)$; otherwise, go to step 2.

We could say that the decision/control vector $\boldsymbol{u}^*(N \times C)$ is the Nash equilibrium if, for each controller $i \in N$, $\boldsymbol{u}_i^*(1 \times C)$ gives the minimum TTS for all players, provided that u_{-i}^* (the decision variables of other controllers) are fixed at their optimum values, i.e.,

$$\underline{\boldsymbol{u}}_i^* \in \arg\min J\left(\underline{\boldsymbol{u}}_i^*, \underline{\boldsymbol{u}}_{-i}^*\right).$$

This means that none of the controllers may change its control value to get a lower TTS, which is the condition of the Nash equilibrium.

In this paper, the SFP algorithm in the MPC framework is designated as the distributed optimization framework (DOF), whereas the conventional nondecomposed optimization is called the centralized optimization framework (COF).

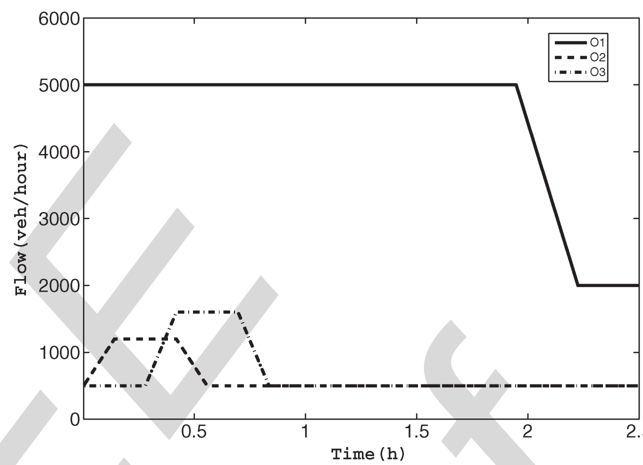

Fig. 4. Demand profiles for all the origins (O1, O2, O3).

VII. Case Study

This section presents the results of a simulation case study performed on a benchmark network. The performance of the proposed algorithm is demonstrated by comparing the achieved TTS values using the DOF and COF, as well as the computational time for the DOF and COF.

A. Network Topology

To assess the performance of the proposed approach, we conducted a series of simulations on a freeway network under three control options, namely, no control, COF, and DOF (the proposed method). The network consists of three origins, including a main stream and two on-ramps. O_1 is the main origin connected to link L_1. The freeway section is 10 km long and is divided into ten segments of equal length (see Fig. 3). The freeway link L_1 has three lanes with a total capacity of 6000 veh/h. The last two segments of link L_1 (segments 3 and 4) are equipped with VMS, where speed limits are applied. AQ1 At the end of link L_1, a single-lane metered on-ramp (O_2) with a capacity of 2000 veh/h is attached. The studied freeway follows via link L_2 with three lanes and four segments to link L_3. At the end of link L_2, another single-lane metered on-ramp (O_3) with a capacity of 2000 veh/h is attached. The studied freeway follows via link L_3 with three lanes and two segments to destination D_1.

To prevent the spill-back of queue to the surface street, we limit the maximum queue length at O_2 and O_3 to 150 and

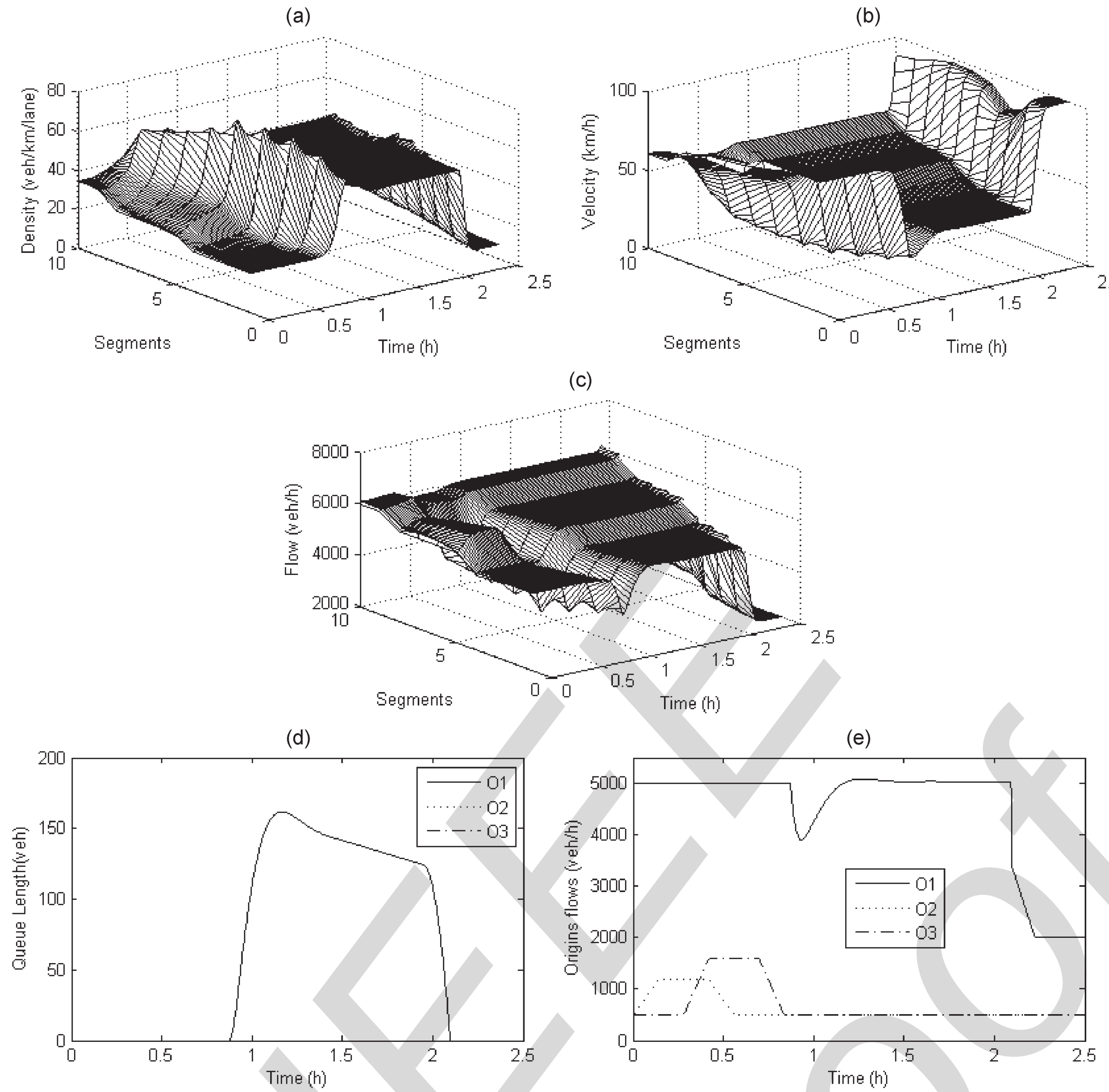

Fig. 5. Simulation results for the no-control case. (a) Segment traffic density. (b) Segment traffic speed. (c) Segment traffic flow. (d) Origin queue length. (e) Origin flow.

80 vehicles, respectively. The network parameters are the same as the parameters used in [23], i.e.,

$$T = 10 \text{ s}, \qquad \tau = 18 \text{ s}$$
$$\kappa = 40 \text{ veh/lane/km}, \qquad \vartheta = 60 \text{ km}^2/\text{h}$$
$$\rho_{\max} = 180 \text{ veh/lane/km}, \qquad a_1 = a_2 = 1.867$$
$$\rho_{\text{crit}} = 33.5 \text{ veh/lane/km}, \qquad V_{\text{free}} = 102.$$

In addition, we assumed that the drivers would obey the control speed displayed by speed limiters ($\alpha = 0$).

The demand profiles from the origins are shown in Fig. 4. The METANET model and the underlying optimization framework are implemented within the MATLAB software.

B. Simulation Results

In the no-control case, when the traffic demands increase in on-ramps 1 and 2, congestion occurs and propagates through links 1 and 2 (see Fig. 5). Consequently, the density on the main stream increases, and a long queue (approximately 150 vehicles) is formed at O_1. In this case, the TTS is 3109 veh.h.

For the MPC system, the optimal prediction and control horizons were found to be approximately 48 and 36 steps, corresponding to 8 and 6 min, respectively. The time step for control updates was set to 1 min, which means that every minute, optimal control must be computed and applied to the traffic system. The simulation results for MPC with COF are shown in Fig. 6. The speed limits reduced the inflow and density of the critical segment, which resulted in a higher outflow. The TTS under this control was 2796 veh.h, which showed 10.06% improvement compared with the no-control case.

The results of the DOF case with the same control parameters used for the previous case are shown in Fig. 7. The TTS in this case was 2605 veh.h, which had an improvement of 16.21% compared with the no-control case and 6.15% to the COF. This result indicates that the DOF could substantially improve the network performance compared with the COF.

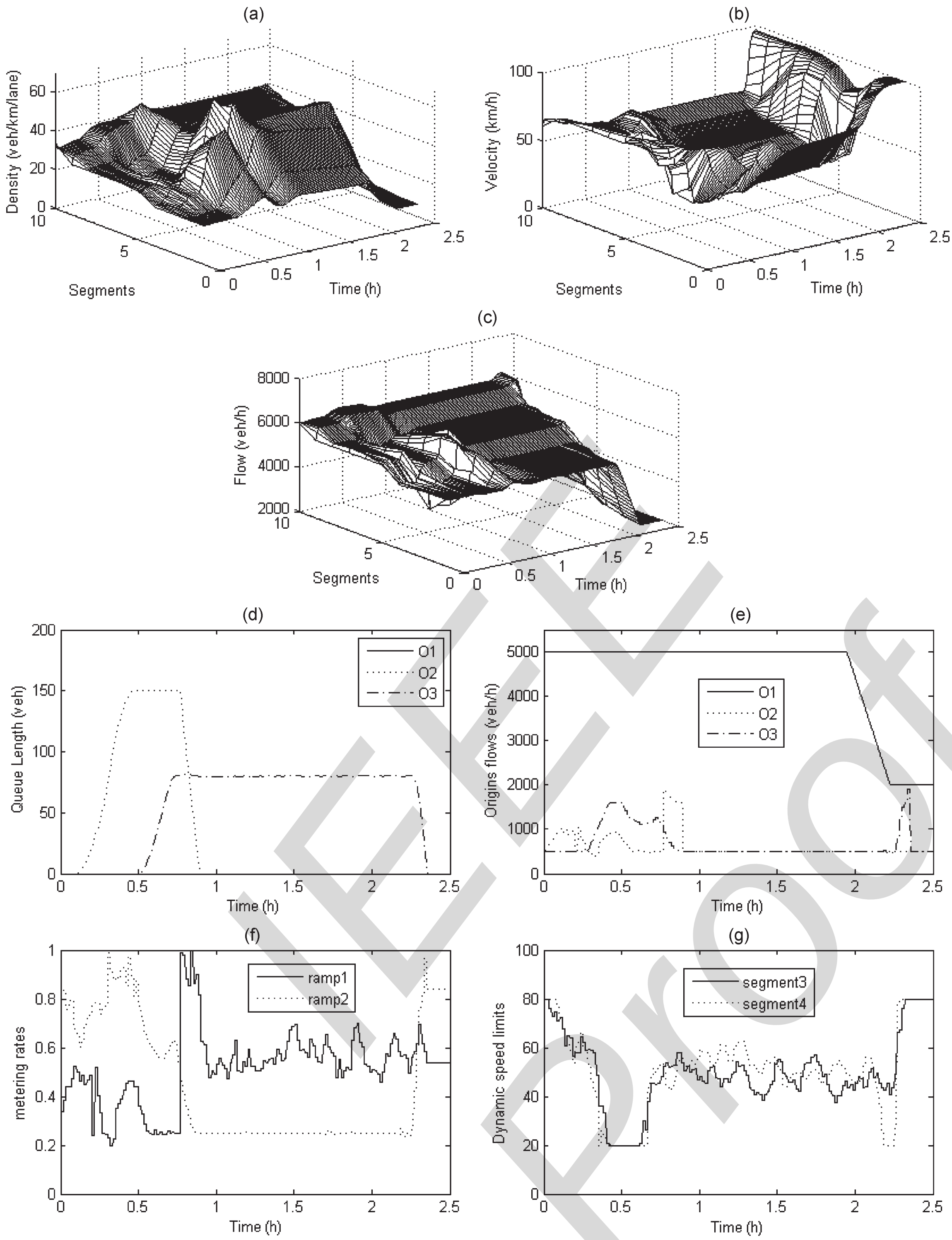

Fig. 6. Simulation results for the COF case. (a) Segment traffic density. (b) Segment traffic speed. (c) Segment traffic flow. (d) Origin queue length. (e) Origin flow. (f) Optimal ramp metering rates. (g) Optimal speed limit values.

Fig. 8 shows the optimal TTS at each control step for the COF and DOF approaches. It can be seen that during the congested period when the control measures are in effect, the TTS values for the DOF case are smaller than those for COF, which results in a better overall performance. This may also be explained by the formation of queues in on-ramps 1 and 2 for two cases. In the COF, the proposed control has used the capacity of the second on-ramp (80 vehicles) for most of the 2.5-h simulation time, whereas in the DOF, the capacity of the first on-ramp (150 vehicles) has mainly been used. These results showed that keeping the vehicles in the first on-ramp has more influence on reducing the TTS. Although no general statement can be made to explain this suboptimal solution achieved by COF, one possible explanation is that, in the COF, a larger search space has to be explored, which degrades the performance of the optimization method. In

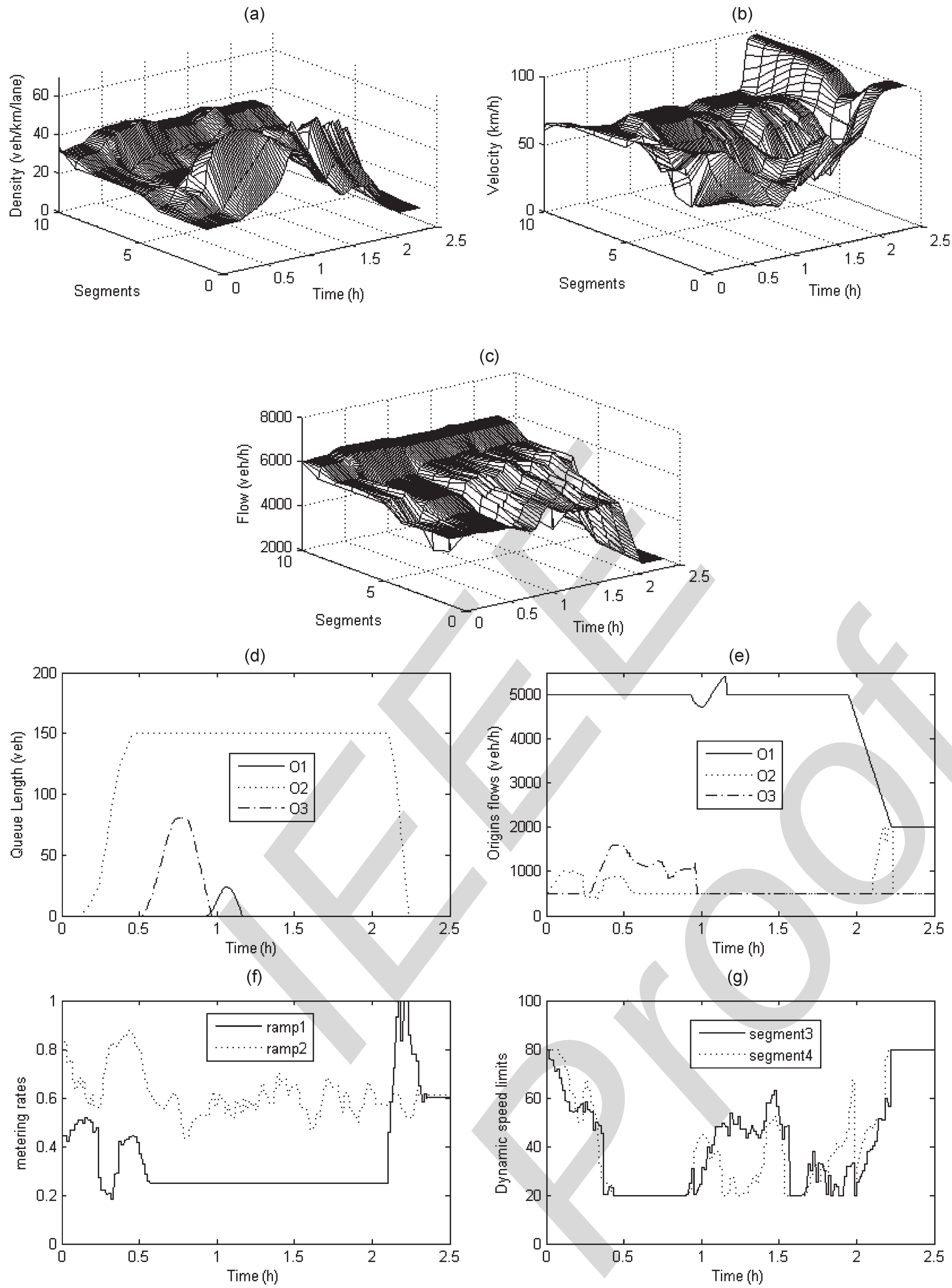

Fig. 7. Simulation results for the DOF case. (a) Segment traffic density. (b) Segment traffic speed. (c) Segment traffic flow. (d) Origin queue length. (e) Origin flow. (f) Optimal ramp metering rates. (g) Optimal speed limit values.

contrast, the DOF keeps the dimension of the decision variables fixed.

In Fig. 9, a sample evolution of the best-reply convergences to the Nash equilibrium value is presented. The results depict that in a few iterations (seven iterations), the optimal TTS value is reached by all players (controllers).

It should be mentioned that our simulation was performed on a single CPU, whereas in real-time control applications, parallel CPUs could be utilized. Therefore, if we assume equal computational time for each player in the proposed simulation, then the total computational time with multiple CPUs would be one fourth of the computation time with a single CPU. In

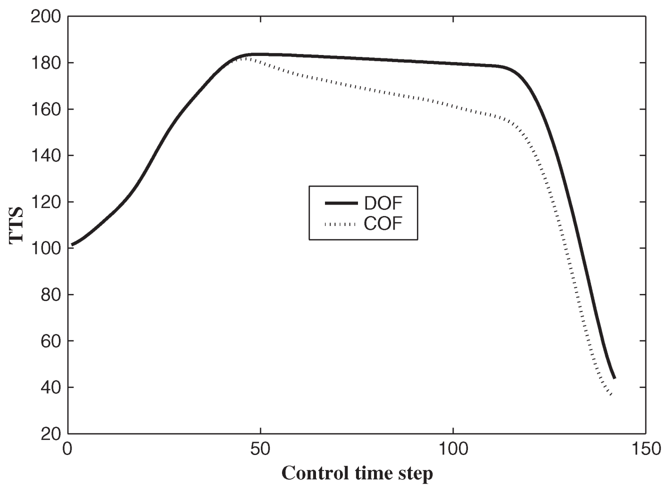

Fig. 8. Optimal TTS for the COF and DOF cases at each control step (in veh.h).

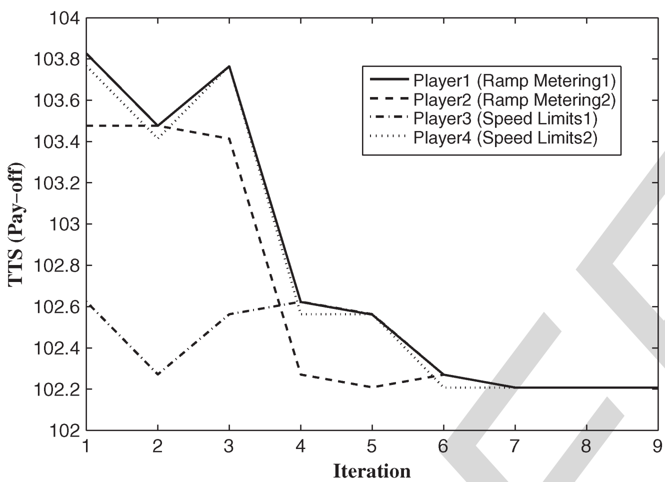

Fig. 9. Evolution of the best-reply convergence.

Fig. 10, the computational time for each control step (on a Pentium IV 3-GHz processor workstation) is plotted for both cases. The average computation time to find the optimal solution in the DOF case was near 20 s and, in the worst case, was less than 60 s, which is the control time step, whereas for the COF, the average time was close to 102 s. Furthermore, it should be noted that the computational time for the DOF approach appeared to grow slowly as the number of control variables increases. This time could further be controlled through parallel implementation.

This improvement in computation time is relative, which means that this time reduction is comparable when an identical software language and optimization algorithm are used for the implementation of the no-control, COF, and DOF cases. Any other implementation of the system in different programming environment or with different optimization algorithm may lead to higher or lower computation time, but the relative time reduction is expected to be the same.

VIII. Conclusion and Future Work

In this paper, a game-theory-based approach has been introduced to address the computational complexity of the integrated and coordinated freeway network-control problem by employing distributed controllers. The proposed method was applied to the problem of optimal ramp metering and variable speed limits in an MPC framework. Based on the simulation results, the proposed method (DOF) achieved better performance in terms of both solution quality and computation time than those for COF. Because of the parallel nature of its solution process, the proposed algorithm can be implemented in parallel in multiple CPUs, making it potentially feasible for real-time implementation in large-size freeway networks.

For future works, we will be focusing on testing the proposed method for larger networks, including more traffic controllers, to investigate changes in the convergence process as the number of traffic controllers increases.

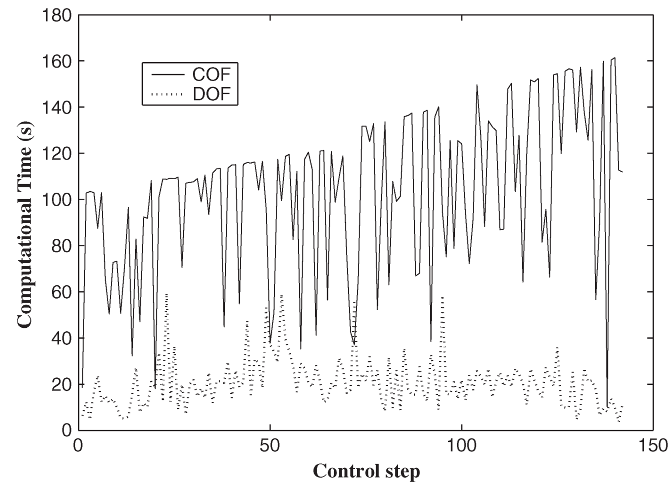

Fig. 10. Computation time for the COF and DOF simulations at each control step (in seconds).

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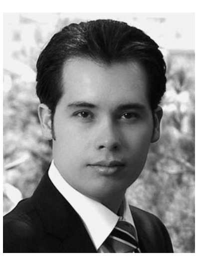

Amir Hosein Ghods (S'07) received the B.Sc. degree in electrical engineering from K. N. Toosi University of Technology, Tehran, Iran, in 2004 and the M.A.Sc. degree in electrical engineering from the University of Tehran in 2008. He is currently working toward the Ph.D. degree in transportation engineering with the University of Waterloo, Waterloo, ON, Canada.

He has provided technical service for the Road Maintenance and Transportation Organization and Tehran Traffic Control companies in Iran. His research interests lie in the area of intelligent transportation systems, traffic control, driving behavior modeling and simulation, and road safety.

Dr. Ghods is a member of the IEEE Intelligent Transportation System Society and the Institute of Transportation Engineers.

Liping Fu is a Professor with the Department of Civil and Environmental Engineering, University of Waterloo, Waterloo, ON, Canada. He has a track record of research contributions to the areas of intelligent transportation systems, traffic engineering, public transportation, winter road maintenance, and road safety, including over 80 journal and conference publications, five software packages for transportation applications, and one patent. He is the developer of a state-of-the-art paratransit scheduling system and two simulation software packages that can be used to evaluate transit performance under realistic operating conditions and management strategies. With his particular interest in integrating results of leading-edge academic research with practical applications, he continues to provide technical services for many transportation agencies, including the Ontario Ministry of Transportation, Transport Canada, the City of Edmonton, ITS Canada, and the U.S. Department of Transportation.

Dr. Fu is the Chair of Transportation Division of the Canadian Society of Civil Engineering, a member of the Transportation Research Board's Paratransit Committee, a member of the Intelligent Transportation Systems Society of Canada, and a member of Canadian Urban Transit Association.

Ashkan Rahimi-Kian (SM'08) received the B.Sc. degree in electrical engineering from the University of Tehran, Tehran, Iran, in 1992 and the M.S. and Ph.D. degrees in electrical engineering from Ohio State University, Columbus, in 1998 and 2001, respectively.

He was the Vice President of Engineering and Development with Genscape, Inc., Louisville, KY, from September 2001 to October 2002 and a Research Associate with the School of Electrical and Computer Engineering (ECE), Cornell University, Ithaca, NY, from November 2002 to December 2003. He is currently an Associate Professor of electrical engineering (Control and Intelligent Processing Center of Excellence) with the School of ECE, College of Engineering, University of Tehran. His research interests include bidding strategies in dynamic energy markets, game theory and learning, intelligent transportation systems, decision making in multiagent stochastic systems, stochastic optimal control, dynamic stock market modeling and decision making using game theory, smart grid design, operation and control, estimation theory and applications in energy and financial systems, risk modeling, and management in energy and financial systems.

AUTHOR QUERIES

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AQ3 = Please provide educational background for L. Fu.

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An Efficient Optimization Approach to Real-Time Coordinated and Integrated Freeway Traffic Control

Amir Hosein Ghods, *Student Member, IEEE*, Liping Fu, and Ashkan Rahimi-Kian, *Senior Member, IEEE*

***Abstract*—This paper tackles the problem of real-time optimal control of traffic flow in a freeway network deployed with coordinated and integrated traffic controllers. One promising approach to this problem is casting the underlying dynamic control problem in a model predictive framework. The challenge is that the resulting optimization problem is computationally intractable for online applications in a network with a large number of controllers. In this paper, a game-theoretic approach with distributed controllers is proposed to address the foregoing issue. The efficiency of the proposed method is tested for a coordinated ramp metering and variable-speed limit control applied to a stretch of freeway network. The parallel nature of the optimization algorithm makes it suitable for solving large-scale problems with high accuracy. The speed and accuracy of the proposed solution approach are examined and compared with that of the conventional optimization method in a case study to demonstrate its superior performance.**

***Index Terms*—Distributed controllers, game theory, model predictive control (MPC), parallel optimization, ramp metering, speed limit control.**

I. Introduction

Several methods have been developed to improve the performance of freeway networks. Among them, control strategies such as ramp metering, speed limits, and route recommendation are recognized as the most effective ways to relieve the freeway traffic congestion. Furthermore, the latest advances in computers and communication technologies have made it feasible to implement network-wide multiple traffic control systems, as opposed to single local control schemes. Intuitively, for a given traffic network, more controllers could result in better performance. Nevertheless, for a network-wide implementation, the amount of data and the computational complexity of the underlying control algorithms quickly increase as the number of control measures increases. Therefore, in general, there exists a tradeoff between the quality of the control method and the amount of information and computational resources required to achieve that quality.

Traffic control strategies can generally be classified into three categories. The first category consists of offline or open-loop strategies, in which only historical data are used in deriving the controls. A good example of open-loop strategies is the fixed-time ramp metering [1], in which the control strategies are predetermined for a particular time of day by solving a linear programming problem based on historical demand. A more sophisticated strategy in this category is the nonlinear optimal ramp-metering method [2], which attempts to minimize an objective function for the whole network. One of the major drawbacks of this control strategy is its high sensitivity to inaccuracies in the predicted traffic demands, traffic patterns, and incidents.

The second category contains the reactive or close-loop methods, which derive the control decisions based on real-time data from traffic sensors such as inductive loop detectors. Generally, this type of controller aims at keeping the freeway conditions as close to a prespecified target state as possible. Reactive ramp metering algorithms such as demand–capacity strategy [3] and ALINEA [4] are popular in this category. These controls do not incorporate any systematic optimization procedure to directly minimize the objective function and are mostly heuristic in nature, and their performance depends on the appropriate selection of the control parameters. Reference [5] provides a comprehensive review of the various ramp-metering methods in these two categories.

The third category includes control strategies commonly called proactive or predictive control methods that make use of both offline and online information to predict the future state of the underlying network and then control the system accordingly. The goal of these strategies is to find the optimal control over a given horizon based on a predefined objective function. It operates in a feedback adaptive fashion by which it takes new observed states and disturbances into account through a prediction model. These control methods are commonly referred to as receding horizon control or model predictive control (MPC). The MPC has been applied in ramp metering [6], variable speed limits [7], combined ramp metering and variable speed limit control [8], and combined dynamic route guidance and ramp-metering control [9].

Despite the obvious advantages of online strategies with optimization frameworks, such as the MPC, they have the drawback that their computational complexity quickly increases by the number of control inputs. This is particularly problematic for

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A. H. Ghods was with the Control and Intelligent Processing Center of Excellence, School of Electrical and Computer Engineering, College of Engineering, University of Tehran, Tehran 14395-515, Iran. He is now with the Transportation Systems Research Group, Department of Civil and Environmental Engineering, University of Waterloo, Waterloo, ON N2L 3G1, Canada (e-mail: ahghods@uwaterloo.ca).

L. Fu is with the Transportation Systems Research Group, Department of Civil and Environmental Engineering, University of Waterloo, Waterloo, ON N2L 3G1, Canada (e-mail: lfu@uwaterloo.ca).

A. Rahimi-Kian is with Control and Intelligent Processing Center of Excellence, School of Electrical and Computer Engineering, College of Engineering, University of Tehran, Tehran 14395-515, Iran (e-mail: arkian@ut.ac.ir).

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traffic-control systems where a closed-form optimal control signal may not explicitly be derived, and for each control interval, an online nonlinear programming technique must be implemented. For instance, Di Febbraro *et al.* [10] proposed to apply artificial neural networks as an offline control for optimal freeway traffic control instead of using online optimization for their receding horizon approach because they found that the dynamics of the system change faster than the speed of the computing system. In another case [8], it was suggested that a hierarchical control scheme be tested that was decomposing the large traffic network into small subnetworks with minimum interaction and then solving each problem locally. In [11], a hierarchical control structure is proposed to the coordinated ramp-metering problem arising in the Amesterdam ring road. The problem was formulated with a nonlinear macroscopic traffic model. The solution method proposed in that work was claimed to be fast enough for real-time implementation; however, it is unknown whether this solution approach could be extended to solve problems with more sophisticated controllers (e.g., speed limits) and input/state constraints. Despite the computational challenges, the potential of online control strategies like the MPC is very promising, and the remaining challenge is to develop a solution method that can feasibly be implemented in a real-world setting.

In this paper, we consider the problem of applying the MPC control framework to the congestion control problem of a freeway network equipped with ramp metering and variable speed limits. A solution algorithm from game theory is proposed to find the optimal solutions for the optimization part of the MPC, which has the potential to make the real-time congestion control computationally tractable even for large traffic networks. A macroscopic traffic flow model is used as the prediction model of the real traffic system. This paper is organized as follows: In Section II, the problem description is presented. In Section III, the basics of the MPC are introduced. In Section IV, the traffic flow model (prediction model) is introduced. In Section V, the problem formulation is proposed. The game-theoretic approach is explained in Section VI. The proposed method is applied to a benchmark problem in Section VII. Finally, conclusions are stated in Section VIII.

II. Integrated and Coordinated Control Problem

We consider the problem of finding the best control settings for a group of controllers in a traffic network consisting of a set of ramp meters and variable speed limit signs. The control objective is to minimize the system-wide total time spent (TTS) by all vehicles in the freeway network. Ramp metering is the most widely used freeway traffic-control method around the world. However, this method will lose its effectiveness as the congestion level increases. Changing the speed limit through variable speed limit signs could partially address this issue and improve the effectiveness of the ramp-metering system, as shown in [8]. The speed limiters located just before the bottleneck on-ramp can help reduce the outflow of controlled segments so that there will be some space left to accommodate the traffic from the on-ramp. This way, the traffic flow in the on-ramp area could be kept near the capacity, and the duration of breakdowns could be reduced. Therefore, a combination of ramp metering and variable speed limit control has the potential to achieve better performance than when they are implanted separately.

Coordination among different controllers that work together is an essential task. For instance, a controller at one spot of a freeway network may mitigate a local congestion problem but may induce congestion at another location on the freeway. Besides using the global data, the prediction of network evolution could be valuable since the effect of control can be seen after a time delay.

As the number of ramp meters and speed control limits increases, the size of the solution vector grows rapidly. For example, to find an optimal solution for N controllers including ramp meters and speed limiters using the MPC approach, (which will be explained in the next section), every controller must find C optimal values at each control time step. Therefore, the solution to the optimal control problem is an $N \times C$ variable matrix. If the problem is formulated as an integer-programming problem with S discrete permissible values for each $N \times C$ variable matrix, then $S^{N\times C}$ values have to be enumerated and evaluated to find the global optimal solution. Although the problem could also be formulated as a continuous nonlinear programming problem, the resulting problem is likely to be nonconvex in nature in that finding the global optimum solution would require an exhaustive search of the whole solution space.

III. Model Predictive Control

The MPC is an advanced control framework that was originally developed for industrial process control (see [12] and [13]). The MPC is a distinguished control model in terms of its capability to deal with various system constraints in an optimization framework. The core idea of the MPC is its use of a dynamic model to predict the future behavior of the system at each optimization step. The goal is to find the desired control inputs such that a predefined objective function is minimized or maximized. In this paper, we have utilized MPC as an online method to optimally control coordination of speed limits and ramp metering with the objective of minimizing the TTS with system states being predicted by a macroscopic freeway model. The following section provides a brief description of the MPC framework introduced in [14].

We consider a control system with N controllers over a specific time horizon. The time horizon is divided into P large control intervals, each subdivided into M small intervals (called system simulation steps). It is assumed that over each control interval, the control variables are kept the same, whereas the system state changes by the simulation step. Let k_c be the index for large intervals ($k_c = 1, 2, \ldots, P$) and k for all the subintervals ($k = 1, 2 \ldots, MP$). The transition of the system state can be expressed as follows:

$$\boldsymbol{x}(k+1) = f\left(\boldsymbol{x}(k), \boldsymbol{u}(k), \boldsymbol{d}(k)\right)$$

where $\boldsymbol{x}(k)$, $\boldsymbol{u}(k)$, and $\boldsymbol{d}(k)$ are vectors representing the system state, the control input, and the disturbance at time k. At each control step k_c, a new optimization is performed to compute the optimal control decisions, e.g.,

$$\boldsymbol{u}(k_c) = \begin{bmatrix} u_1(k_c) & u_1(k_c+1) & \cdots & u_1(k_c+P-1) \\ & & \vdots & \\ u_N(k_c) & u_N(k_c+1) & \cdots & u_N(k_c+P-1) \end{bmatrix}$$

for the time period of $[1.2, \ldots, P]$, in which P is the prediction horizon.

To reduce the computational complexity, a control horizon $C(C < P)$ is usually defined to represent the time horizon over which the control signal is considered to be fixed, i.e.,

$$u(k_c) = u(C-1) \; for \; k_c > C.$$

Therefore, for N controllers, the $N \times C$ vector of optimal controls would be

$$\boldsymbol{u}^*(k_c) = \begin{bmatrix} u_1^*(k_c) & u_1^*(k_c+1) & \cdots & u_1^*(k_c+C-1) \\ & & \vdots & \\ u_N^*(k_c) & u_N^*(k_c+1) & \cdots & u_N^*(k_c+C-1) \end{bmatrix}.$$

Only the first optimal control signal $\boldsymbol{u}_i^*(k_c)$, $i = 1, 2, \ldots, N$ (first column) is applied to the real system, and after shifting the prediction and control horizon one step forward with the current observed states of the real system to the model, the process is repeated. This feedback is necessary to correct any prediction errors and system disturbances that may deviate from model prediction. Since we have to work with a nonlinear system (traffic model), in each control time step k_c, a nonlinear programming has to be solved to find the $N \times C$ optimal solutions before reaching the next control time step $(k_c + 1)$.

It should be pointed out that the control parameters P and C need to be selected appropriately. Choosing a large prediction and control horizon will increase the computational demands due to the increased number of optimization variables. On the other hand, using a short prediction and control horizon may turn the control strategy into a reactive model and thus degrade its effectiveness.

In the following sections, we introduce how the system state equations are modeled using a dynamic traffic flow model and how the MPC can be cast into a game-theoretical framework and solved efficiently.

IV. Traffic-Flow Model

The traffic-flow model adopted here is the destination independent METANET model (see [2] for more details) together with the extended model for speed limits presented in [8].

The METANET is a macroscopic traffic model that is discrete in both space and time. The model represents the network by a directed graph with a set of links corresponding to freeway stretches and a set of nodes, as illustrated in Fig. 1. Each link has uniform characteristics i.e., no on-ramp or off-ramp and no major changes in geometry. The nodes of the graph are placed between links, where the major change in road geometry occurs, such as on-ramps and off-ramps. A freeway link (m) is divided into (N_m) segments (indexed by i) of length $(l_{m,i})$ and by the number of lanes (n_m). Each segment (i) of link (m) at time instant $t = kT$, where T is the time step used for simulation, and $k = 0, \ldots, K$, is macroscopically characterized by its *traffic density* $\rho_{m,i}(k)$ (in vehicles per lane per kilometer), *mean speed* $v_{m,i}(k)$ (in kilometers per hour), and *traffic volume* $q_{m,i}(k)$ (in vehicles per hour). Table I describes the notations related to the METANET model.

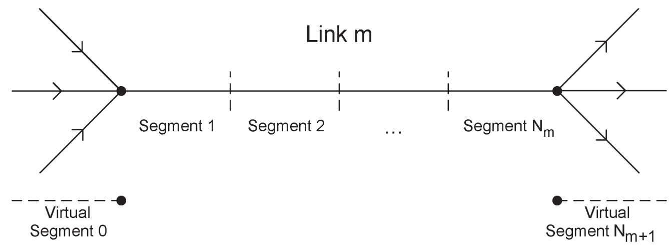

Fig. 1. METANET model. Link and node configuration.

The traffic stream models that capture the evolution of traffic on each segment at each time step are shown in (1)–(8) (see Table II). The node equations that represent the relation between connected links are given in (9)–(12) (see Table III), which show how the entering traffic flow to a node is distributed among the emanating links.

Using the aforementioned equations, the nonlinear traffic dynamics can be expressed as follows:

$$\boldsymbol{x}(k+1) = f\left(\boldsymbol{x}(k), \boldsymbol{u}(k), \boldsymbol{d}(k)\right) \tag{13}$$

where $\boldsymbol{x}(k)$ is the state vector of the system, that is, flow rate $(q_{m,i}(k))$, speed $(v_{m,i}(k))$, density $(\rho_{m,i}(k))$, and queue length of origins $w_o(k)$; $\boldsymbol{u}(k)$ is the vector of control inputs, including the ramp metering rates and the speed limits; and $\boldsymbol{d}(k)$ is the disturbance vector at simulation step k.

Based on $\boldsymbol{x}(k)$, $\boldsymbol{u}(k)$, and $\boldsymbol{d}(k)$, the future evolution of the traffic system $[\widehat{\boldsymbol{x}}(k+1), \ldots, \widehat{\boldsymbol{x}}(k+MP-1)]$ can be predicted by the METANET model.

V. Problem Formulation

With the definitions and system state equations introduced in the previous section, we can now present the formulation of the MPC optimization problem. The optimal control problem includes the following two sets of decision variables:

1) $v_i(j)$: variable speed limits for $j \in [k, \ldots, k+C-1]$ and $i \in I_{\text{speed}}$, where I_{speed} is the set of speed limits that are presented in the freeway network;
2) $r_o(j)$: ramp-metering rates for $j \in [k, \ldots, k+C-1]$ and $o \in O_{\text{ramp}}$, where O_{ramp} is the set of controlled on-ramps where ramp metering is presented.

TABLE I
NOTATIONS USED IN THE METANET MODEL

m, μ	Link index
i	Segment index
T	Simulation step size
k	Time step counter
$\rho_{m,i}(k)$	Density of segment i of freeway link m (veh/km/lane)
$v_{m,i}(k)$	Speed of segment i of freeway link m (km/h)
$q_{m,i}(k)$	Flow of segment i of freeway link m (veh/h)
N_m	Number of segments in link m
n_m	Number of lanes in link m
$l_{m,i}$	Length of segment i in link m (km)
τ	Time constant of the speed relaxation term (h)
κ	Speed anticipation term parameter (veh/km/lane)
υ	Speed anticipation term parameter (km^2/h)
α_m	Parameter of the fundamental diagram
$\rho_{crit,m}$	Critical density of link m (veh/km/lane)
$V(\rho_{m,i}(k))$	Speed of segment i of link m on a homogeneous freeway as a function of the density $\rho_{m,i}(k)$
$\rho_{max,m}$	Maximum density (veh/km/lane) of link m
$v_{free,m}$	Free-flow speed of link m (km/h)
$w_o(k)$	Length of the queue on on-ramp o at the time step k (veh)
$q_o(k)$	Flow that enters into the freeway at time sep k (veh/h)
$d_o(k)$	Traffic demand at origin o at time step k (veh/h)
$r_o(k)$	Ramp metering rate of on-ramp o at time step k
Q_o	On-ramp capacity (veh/h)
δ	Speed drop term parameter caused by merging at an on-ramp
n	Node index
Q_n	Total flow that enters freeway node n (veh/h)
I_n	Set of link indexes that enter node n
O_n	Set of link indexes that leave node n
β_n^m	Fraction of the traffic that leaves node n via link m
$v_{control,m,i}$	Speed limit applied in segment i of link m (km/h)
α	Parameter expressing the disobedience of drivers with the displayed speed limits

The objective function used in this paper is the TTS spent by all vehicles, as defined in

$$\begin{aligned}\mathrm{TTS} &= J(v,r)\\ &= T\sum_{j=k}^{k+P-1}\left\{\sum_{m,i}\rho_{m,i}(j)l_{m,j}n_m + \sum_o w_o(j)\right\}\\ &\quad + \sum_{j=k}^{k+P-1}\left\{\alpha_{\text{ramp}}\sum_{o\in O_{\text{ramp}}}(r_o(j)-r_o(j-1))^2\right.\\ &\qquad\left. + \alpha_{\text{speed}}\sum_{i\in I_{\text{speed}}}\left(\frac{v_i(j)-v_i(j-1)}{v_{\text{free}}}\right)^2\right\}\\ &\quad + \alpha_{\text{queue}}\sum_{o\in O_{\text{ramp}}}(\max(w_o-w_{\max}))^2. \qquad (14)\end{aligned}$$

The first two terms in (14) correspond to the main stream and the origins' queues, respectively. The second and third terms, which are weighted by nonnegative weighting factors, enable the control strategy to penalize abrupt changes in the ramp metering and speed-limit-control decisions, and the last term with a nonnegative weighting factor penalizes queue lengths larger than the on-ramp capacity for keeping the queue lengths within the permissible limit of the on-ramps.

The MPC optimization problem can therefore be formulated as follows in an abbreviated form:

$$\begin{aligned}&\min \quad \{J(\boldsymbol{v},\boldsymbol{r}) : \boldsymbol{v}\in\boldsymbol{V}, \boldsymbol{r}\in\boldsymbol{R}\}\\ &\text{s.t.} \quad \text{Equations (1)–(12)} \qquad (15)\end{aligned}$$

where for N_1 speed limits and N_2 ramp meters, $\boldsymbol{v}(N_1 \times C)$ and $\boldsymbol{r}(N_2 \times C)$ are decision variables, respectively, $(N_1 + N_2 = N)$, and $\boldsymbol{V} \times \boldsymbol{R}$ is the feasible search space. We

TABLE II
LINK EQUATIONS AND DESCRIPTIONS

$q_{m,i}(k)=\rho_{m,i}(k)v_{m,i}(k)n_m$	(1)	Flow-Density-Speed equation
$\rho_{m,i}(k+1)=\rho_{m,i}(k)+\frac{T}{l_{m,i}n_m}[q_{m,i-1}(k)-q_{m,i}(k)]$	(2)	Conservation of vehicles
$v_{m,i}(k+1)=v_{m,i}(k)+\underbrace{\frac{T}{\tau_m}\{V[\rho_{m,i}(k)]-v_{m,i}(k)\}}_{\text{Relaxation Term}}+\underbrace{\frac{T}{l_{m,i}}v_{m,i}(k)[v_{m,i-1}(k)-v_{m,i}(k)]}_{\text{Convection Term}}-\underbrace{\frac{\vartheta_m T}{\tau_m.l_{m,i}}\frac{\rho_{m,i+1}(k)-\rho_{m,i}(k)}{\rho_{m,i}(k)+\kappa_m}}_{\text{Anticipation Term}}$	(3)	Speed dynamic Relaxation Term: drivers try to achieve desired speed $V(\rho)$. Convection Term: Speed decrease or increase caused by inflow of vehicles. Anticipation Term: the speed decrease (increase) as drivers experience the density increase (decrease) in downstream.
$V[\rho_{m,i}(k)]=v_{free,m}\exp\left(-\frac{1}{a_m}\left(\frac{\rho_{m,i}(k)}{\rho_{crit,m}}\right)^{a_m}\right)$	(4)	Speed-Density relation (fundamental diagram)
$w_o(k+1)=w_o(k)+T(d_o(k)-q_o(k))$	(5)	Origins' queueing model
$q_o(k)=\min\left[d_o(k)+\frac{w_o(k)}{T},Q_o.r_o(k),\; Q_o\frac{\rho_{\max,m}-\rho_{m,1}(k)}{\rho_{\max,m}-\rho_{crit,m}}\right]$	(6)	Ramp outflow equation The outflow depends on the traffic condition in the main-stream and also on the metering rate, $r_o(k)\in[0,1]$
$V(\rho_{m,i}(k))=\min\left\{v_{free,m}\exp\left(-\frac{1}{a_m}\left(\frac{\rho_{m,i}(k)}{\rho_{crit,m}}\right)^{a_m}\right),\; (1+\alpha)v_{control,m,i}(k)\right\}$	(7)	Speed limit model The desired speed is the minimum of the speed determined by (4) and the speed limit, which is displayed on the variable message sign (VMS).
$-\frac{\delta T q_o(k)v_{m,1}}{l_{m,i}n_m(\rho_{m,1}(k)+\kappa)}$	(8)	Speed drop caused by merging phenomena. If there is an on-ramp then the term must be added to (3)

TABLE III
NODE EQUATIONS AND DESCRIPTIONS

$Q_n(k)=\sum_{\mu\in I_n}q_{\mu,N_\mu}(k)$	(9)	Total traffic flow enter node n
$q_{m,0}(k)=\beta_n^m(k).Q_n(k)$	(10)	Traffic flow that leaves node n via link m
$\rho_{m,N_{m+1}}(k)=\frac{\sum_{\mu\in O_n}\rho_{\mu,1}^2(k)}{\sum_{\mu\in O_n}\rho_{\mu,1}(k)}$	(11)	Virtual downstream density, when node n has more than one leaving link
$v_{m,0}(k)=\frac{\sum_{\mu\in I_n}v_{\mu,N_\mu}(k).q_{\mu,N_\mu}(k)}{\sum_{\mu\in I_n}q_{\mu,N_\mu}(k)}$	(12)	Virtual upstream speed, when node n has more than one entering link

call the whole decision variable vector $\boldsymbol{u}(N\times C)$, which is as follows:

$$\boldsymbol{u}(k_c)=\begin{bmatrix} v_1(k_c) & v_1(k_c+1) & \cdots & v_1(k_c+C-1)\\ & & \vdots & \\ v_{N_1}(k_c) & v_{N_1}(k_c+1) & \cdots & v_{N_1}(k_c+C-1)\\ r_1(k_c) & r_1(k_c+1) & \cdots & r_1(k_c+C-1)\\ & & \vdots & \\ r_{N_2}(k_c) & r_{N_2}(k_c+1) & \cdots & r_{N_2}(k_c+C-1)\end{bmatrix}.$$

Because of the nonlinearity of the traffic system states (1)–(12) and the objective function, this problem is a nonlinear programming with $N\times C$ decision variables. The problem is commonly solved using sequential quadratic programming (SQP) algorithm [8]. However, the SQP algorithm is viable only for small problems, and its optimality is not guaranteed. Therefore, to find a sufficiently good solution in a reasonable time for this problem, we apply the game theory that has successfully been applied to solve large-size optimization problems in other fields.

VI. Game-Theoretic Approach

The game theory was first introduced in the economy to find the market equilibrium when multiple firms compete with each other to sell or buy some goods. Game theory studies how rational decision makers (players) choose their strategies from the sets of decisions that depend on the strategies of other players. In other words, each player has a payoff function that is affected by the strategy of the player itself and the strategies of other players. There are two types of strategies defined in game theory: 1) If a player has a dominant strategy or knows what his/her opponent will do in the next step, then he/she could take a strategy with probability 1, which is called *pure strategy*. 2) However, in incomplete information games where players do not have dominant strategies or are not sure about the next step decisions of their rivals, they may assign different probabilities to their own and their rivals' decision sets, and their strategy vectors are called *mixed strategies* (for more details regarding game theory and applications, see [15] and [16]).

The basic idea of using game theory in this paper for freeway optimal traffic control is to decompose the whole optimization problem into a number of suboptimization problems with smaller dimensions and to solve them individually but in a coordinated way. This is similar to turning the optimization problem into a sequential and coordinated game that is played by a number of players with identical payoffs. In our case, each of the N controllers in the traffic network is considered as a player in a game, and the TTS of all vehicles in the network is considered the objective function of all the players. Therefore, the optimal coordination of the ramp metering and variable speed limits is presented as a game of identical interests.

Since the players (traffic controllers) decide simultaneously and try to chose their best strategies in response to the predicted strategies of their rivals (other network controllers), the solution vector of such game represents a state called *Nash equilibrium*, in which the players cannot improve their payoffs by changing their strategies unilaterally. The Nash equilibrium solution can be found through a well-known algorithm called fictitious play (FP) [17]. The FP is an interactive process in which the players find their best strategies by predicting the rivals' strategies based on the probability distributions of their past decisions. In general, the FP is not guaranteed to converge to the Nash equilibrium; however, it does converge to the Nash equilibrium in games of identical interest or common objective (in our case TTS) [18]. Virtually, the optimization problems may be viewed as a game of identical objectives in which the Nash solution has some optimality properties; as a result, the FP has recently become increasingly popular as an optimization tool.

The classical form of FP is computationally extensive in practice. Reference [19] proposed a modified form of it called sample FP (SFP) that is similar to the original FP with a difference that the best strategies are computed against a random sample from the history of the past decisions of the rivals instead of the predicted decisions based on their probability distributions. The SFP algorithm is useful to solve the problem of form (15), particularly when the objective function is evaluated through a black-box module requiring significant computational efforts for each function evaluation similar to our case (see [19] for more details). In the SFP method, each player finds its best strategy by assuming that other players play known strategies drawn randomly from the history of their past plays. Therefore, players learn other players' strategies iteratively. The convergence of the SFP with the increasing number of iterations has also been proven in [19]. The SFP algorithm has been applied for solving the dynamic traffic-assignment problem [20], the communication protocol design [21], and the signalized intersection problem [22].

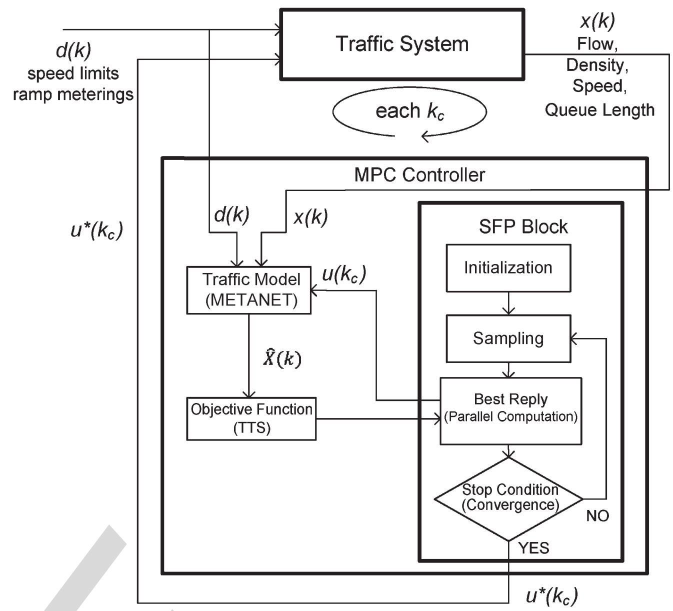

Fig. 2. Schematic diagram of MPC with SFP optimization method.

The SFP algorithm has the following steps, as reported in [22]:

1) Initialization: A set of initial strategies is randomly chosen for each player and stored in the history.
2) Sampling: A strategy arbitrarily drawn from the history of plays for each player with equal probability.
3) Best reply: Each player computes his/her best reply or strategy, assuming that other players play the strategies drawn in the previous step.
4) Store: The best replies obtained in Step 3 are stored in the history of plays.
5) Stop Condition: Check whether the stopping criterion is met (for example, if the solution vector has reached the steady-state Nash equilibrium); if not, then go to Step 2.

The most important feature of the SFP algorithm is that the best-reply computation can be done in parallel for all players simultaneously. This makes the algorithm feasible for parallel implementation, that is, the N, C-dimensional optimization problems can be solved in parallel. It is also possible to decompose the problem into much smaller subproblems by assuming the C control signal of each controller as an individual player. Accordingly, we would have $N \times C$ players, each with a 1-D optimization problem. We omitted this configuration because in this scheme the divergence time associated with $N \times C$ players might have become problematic as the number of controlled inputs would increase. Furthermore, the C-dimensional problem is small enough for our optimization algorithm, and

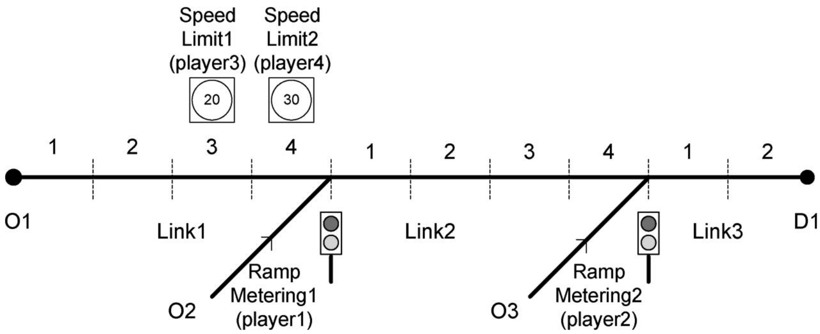

Fig. 3. Benchmark network with two on-ramp metering and two speed limits. Each controller has been considered as a player.

the parameter C does not vary as the number of controller increases.

The SFP algorithm of coordinated ramp metering and variable speed limits in the MPC framework can be presented as follows (see Fig. 2 for the schematic description):

1) Initialization: A set of initial values is randomly chosen for each of the ramp meters and speed limits for a given control horizon (C). $(\boldsymbol{u}_i^{\text{initial}}(1 \times C)$ for $i = 1, \ldots, N)$.
2) Sampling: The control values are arbitrarily drawn from the history of previously stored values for each controller with equal probability (equal to initial values for the first step). $(\boldsymbol{u}_i^{\text{history}}(1 \times C)$ for $i = 1, \ldots, N)$.
3) Optimization: Each controller finds its optimal values by minimizing the objective function of (14) over the prediction horizon, assuming that all the other controllers have taken constant values (drawn from Step 2). The METANET model is utilized as the prediction model and the SQP algorithm as a numerical optimization algorithm to find the optimal controls. $\boldsymbol{u}_i^*(1 \times C)$ for $i = 1, \ldots, N$.
4) Store: The new optimal values obtained in Step 3 are stored in the history of the players' decisions.
5) Stop Condition: Checks whether the convergence of the fitness function for each controller has occurred (i.e., if the steady-state Nash equilibrium has been reached). If yes, then stop and repeat this algorithm for the next iteration $(k+1)$; otherwise, go to step 2.

We could say that the decision/control vector $\boldsymbol{u}^*(N \times C)$ is the Nash equilibrium if, for each controller $i \in N$, $\boldsymbol{u}_i^*(1 \times C)$ gives the minimum TTS for all players, provided that u_{-i}^* (the decision variables of other controllers) are fixed at their optimum values, i.e.,

$$\underline{\boldsymbol{u}}_i^* \in \arg\min J\left(\underline{\boldsymbol{u}}_i^*, \underline{\boldsymbol{u}}_{-i}^*\right).$$

This means that none of the controllers may change its control value to get a lower TTS, which is the condition of the Nash equilibrium.

In this paper, the SFP algorithm in the MPC framework is designated as the distributed optimization framework (DOF), whereas the conventional nondecomposed optimization is called the centralized optimization framework (COF).

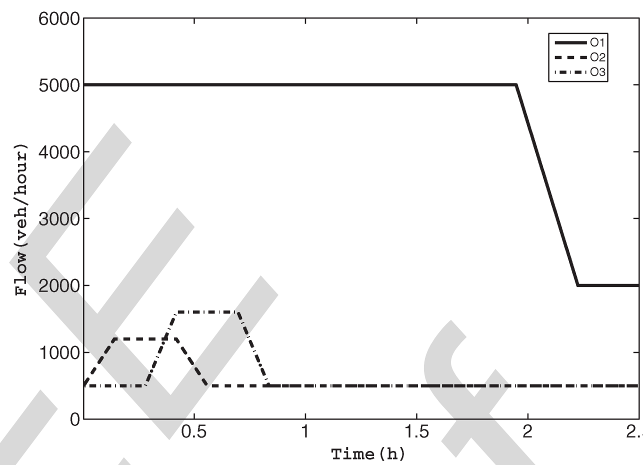

Fig. 4. Demand profiles for all the origins (O1, O2, O3).

VII. Case Study

This section presents the results of a simulation case study performed on a benchmark network. The performance of the proposed algorithm is demonstrated by comparing the achieved TTS values using the DOF and COF, as well as the computational time for the DOF and COF.

A. Network Topology

To assess the performance of the proposed approach, we conducted a series of simulations on a freeway network under three control options, namely, no control, COF, and DOF (the proposed method). The network consists of three origins, including a main stream and two on-ramps. O_1 is the main origin connected to link L_1. The freeway section is 10 km long and is divided into ten segments of equal length (see Fig. 3). The freeway link L_1 has three lanes with a total capacity of 6000 veh/h. The last two segments of link L_1 (segments 3 and 4) are equipped with VMS, where speed limits are applied. AQ1 At the end of link L_1, a single-lane metered on-ramp (O_2) with a capacity of 2000 veh/h is attached. The studied freeway follows via link L_2 with three lanes and four segments to link L_3. At the end of link L_2, another single-lane metered on-ramp (O_3) with a capacity of 2000 veh/h is attached. The studied freeway follows via link L_3 with three lanes and two segments to destination D_1.

To prevent the spill-back of queue to the surface street, we limit the maximum queue length at O_2 and O_3 to 150 and

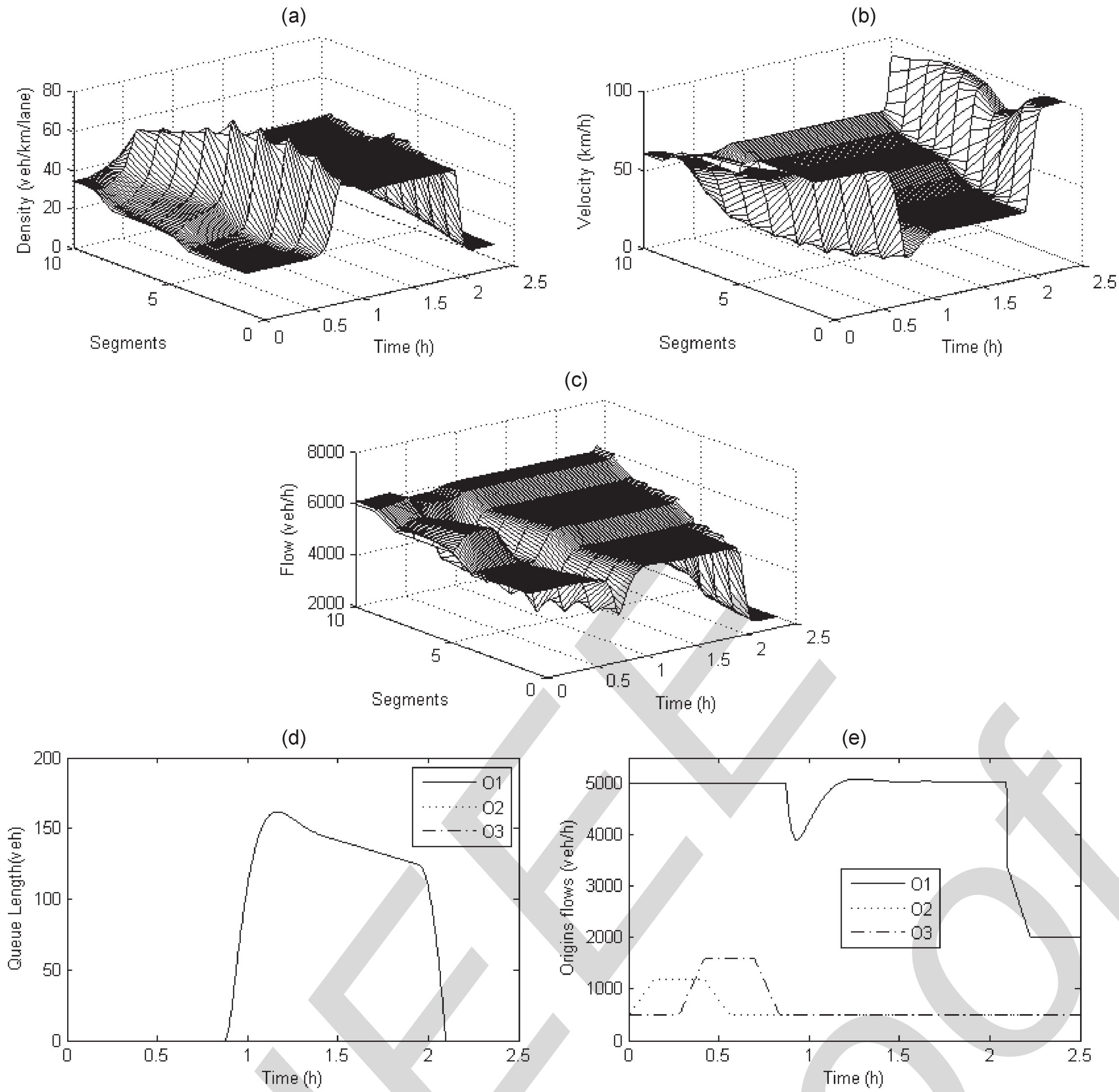

Fig. 5. Simulation results for the no-control case. (a) Segment traffic density. (b) Segment traffic speed. (c) Segment traffic flow. (d) Origin queue length. (e) Origin flow.

80 vehicles, respectively. The network parameters are the same as the parameters used in [23], i.e.,

$$T = 10 \text{ s}, \qquad \tau = 18 \text{ s}$$

$$\kappa = 40 \text{ veh/lane/km}, \qquad \vartheta = 60 \text{ km}^2/\text{h}$$

$$\rho_{\max} = 180 \text{ veh/lane/km}, \qquad a_1 = a_2 = 1.867$$

$$\rho_{\text{crit}} = 33.5 \text{ veh/lane/km}, \qquad V_{\text{free}} = 102.$$

In addition, we assumed that the drivers would obey the control speed displayed by speed limiters ($\alpha = 0$).

The demand profiles from the origins are shown in Fig. 4. The METANET model and the underlying optimization framework are implemented within the MATLAB software.

B. Simulation Results

In the no-control case, when the traffic demands increase in on-ramps 1 and 2, congestion occurs and propagates through links 1 and 2 (see Fig. 5). Consequently, the density on the main stream increases, and a long queue (approximately 150 vehicles) is formed at O_1. In this case, the TTS is 3109 veh.h.

For the MPC system, the optimal prediction and control horizons were found to be approximately 48 and 36 steps, corresponding to 8 and 6 min, respectively. The time step for control updates was set to 1 min, which means that every minute, optimal control must be computed and applied to the traffic system. The simulation results for MPC with COF are shown in Fig. 6. The speed limits reduced the inflow and density of the critical segment, which resulted in a higher outflow. The TTS under this control was 2796 veh.h, which showed 10.06% improvement compared with the no-control case.

The results of the DOF case with the same control parameters used for the previous case are shown in Fig. 7. The TTS in this case was 2605 veh.h, which had an improvement of 16.21% compared with the no-control case and 6.15% to the COF. This result indicates that the DOF could substantially improve the network performance compared with the COF.

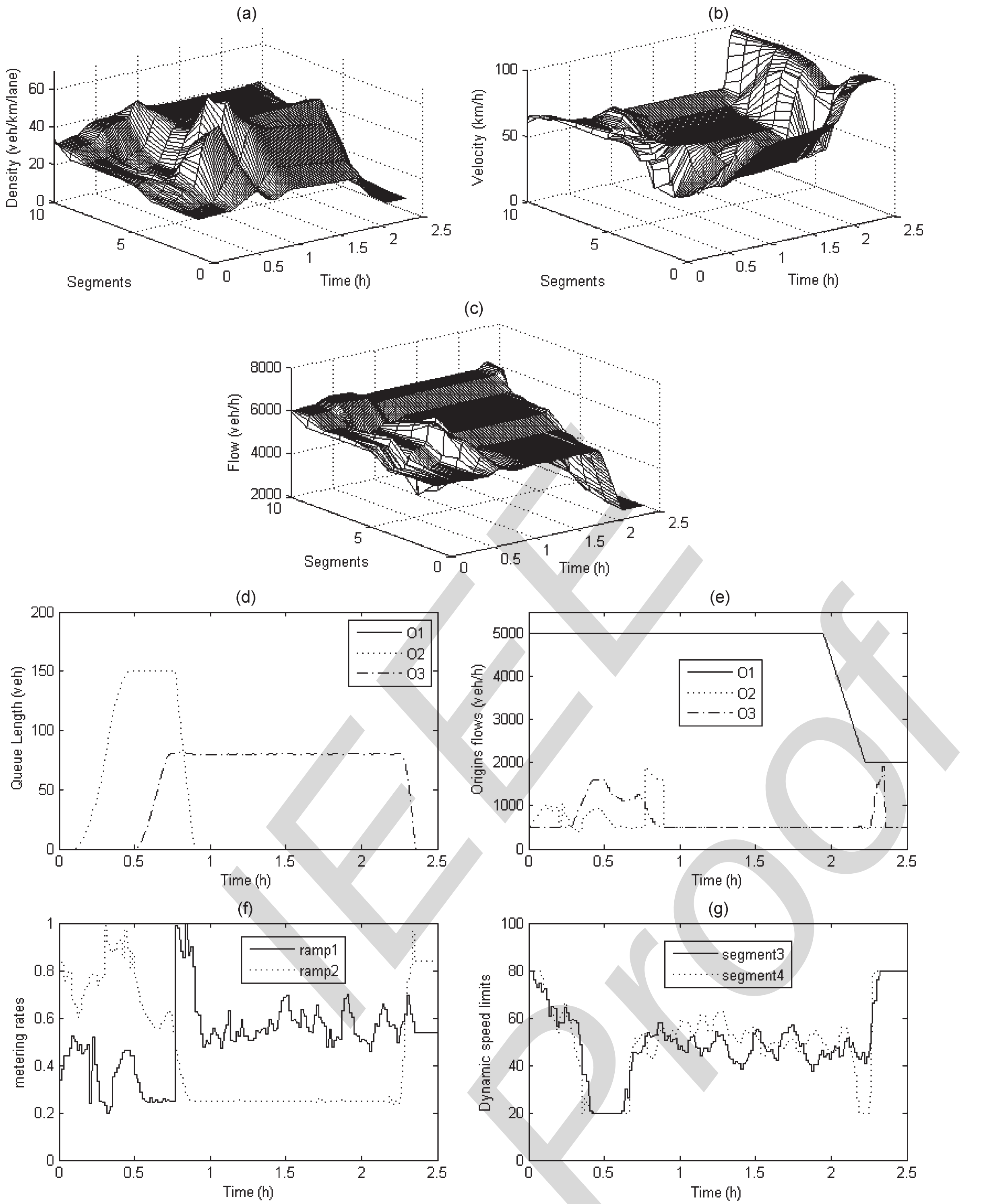

Fig. 6. Simulation results for the COF case. (a) Segment traffic density. (b) Segment traffic speed. (c) Segment traffic flow. (d) Origin queue length. (e) Origin flow. (f) Optimal ramp metering rates. (g) Optimal speed limit values.

Fig. 8 shows the optimal TTS at each control step for the COF and DOF approaches. It can be seen that during the congested period when the control measures are in effect, the TTS values for the DOF case are smaller than those for COF, which results in a better overall performance. This may also be explained by the formation of queues in on-ramps 1 and 2 for two cases. In the COF, the proposed control has used the capacity of the second on-ramp (80 vehicles) for most of the 2.5-h simulation time, whereas in the DOF, the capacity of the first on-ramp (150 vehicles) has mainly been used. These results showed that keeping the vehicles in the first on-ramp has more influence on reducing the TTS. Although no general statement can be made to explain this suboptimal solution achieved by COF, one possible explanation is that, in the COF, a larger search space has to be explored, which degrades the performance of the optimization method. In

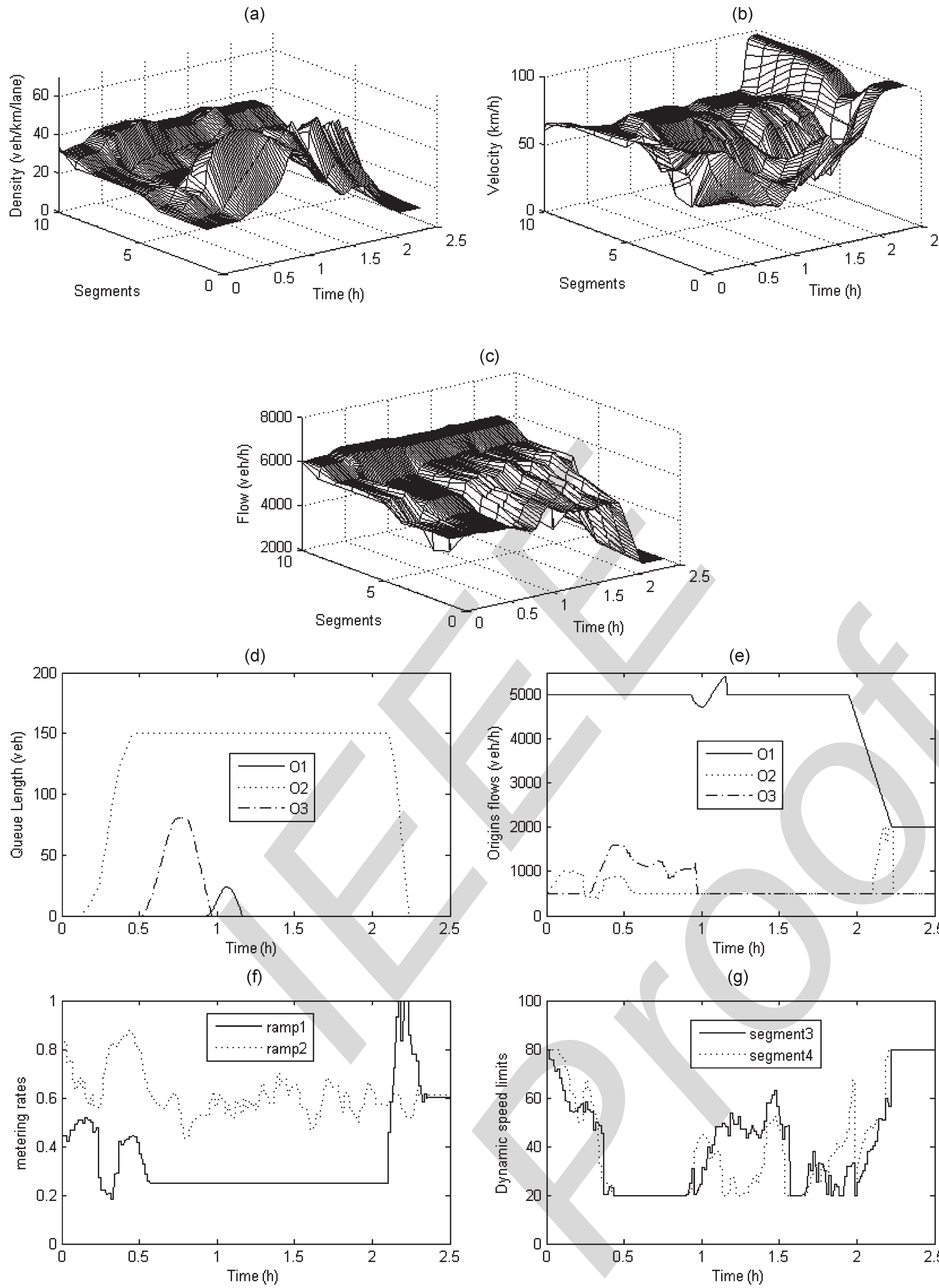

Fig. 7. Simulation results for the DOF case. (a) Segment traffic density. (b) Segment traffic speed. (c) Segment traffic flow. (d) Origin queue length. (e) Origin flow. (f) Optimal ramp metering rates. (g) Optimal speed limit values.

contrast, the DOF keeps the dimension of the decision variables fixed.

In Fig. 9, a sample evolution of the best-reply convergences to the Nash equilibrium value is presented. The results depict that in a few iterations (seven iterations), the optimal TTS value is reached by all players (controllers).

It should be mentioned that our simulation was performed on a single CPU, whereas in real-time control applications, parallel CPUs could be utilized. Therefore, if we assume equal computational time for each player in the proposed simulation, then the total computational time with multiple CPUs would be one fourth of the computation time with a single CPU. In

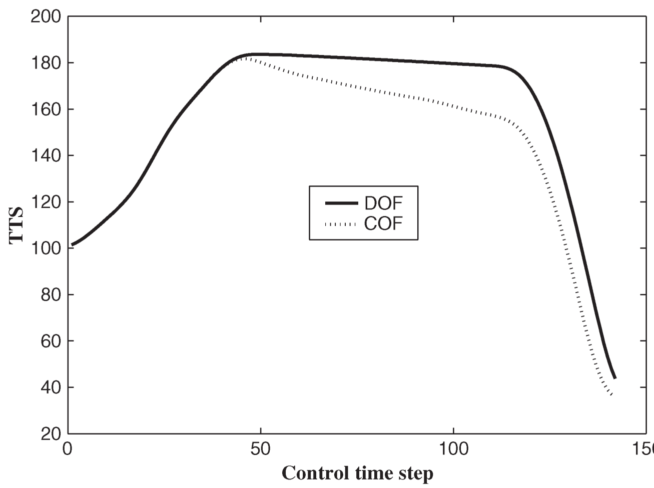

Fig. 8. Optimal TTS for the COF and DOF cases at each control step (in veh.h).

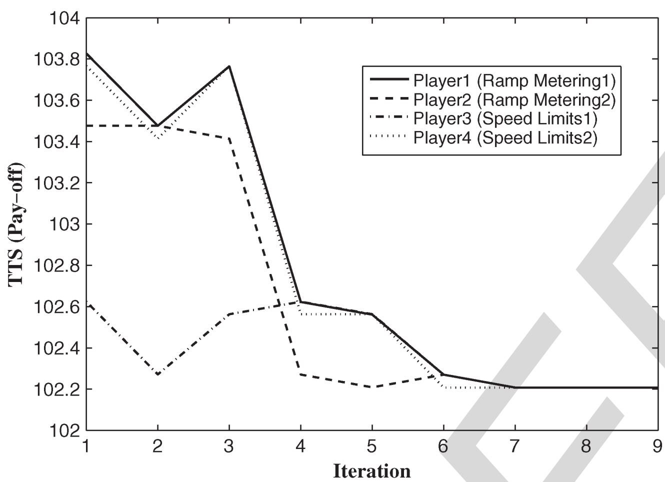

Fig. 9. Evolution of the best-reply convergence.

Fig. 10, the computational time for each control step (on a Pentium IV 3-GHz processor workstation) is plotted for both cases. The average computation time to find the optimal solution in the DOF case was near 20 s and, in the worst case, was less than 60 s, which is the control time step, whereas for the COF, the average time was close to 102 s. Furthermore, it should be noted that the computational time for the DOF approach appeared to grow slowly as the number of control variables increases. This time could further be controlled through parallel implementation.

This improvement in computation time is relative, which means that this time reduction is comparable when an identical software language and optimization algorithm are used for the implementation of the no-control, COF, and DOF cases. Any other implementation of the system in different programming environment or with different optimization algorithm may lead to higher or lower computation time, but the relative time reduction is expected to be the same.

VIII. Conclusion and Future Work

In this paper, a game-theory-based approach has been introduced to address the computational complexity of the integrated

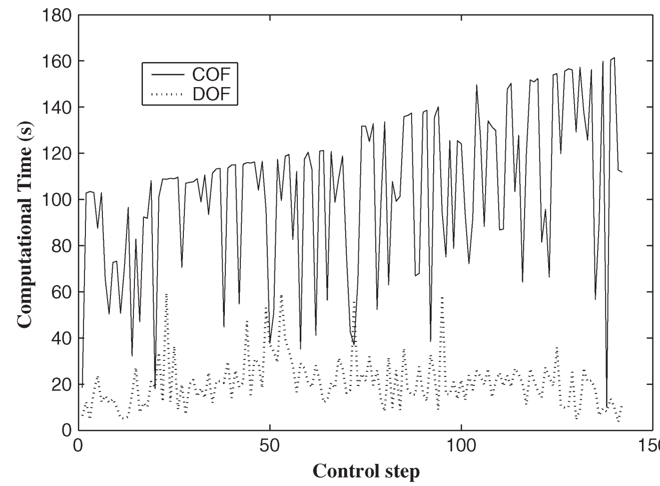

Fig. 10. Computation time for the COF and DOF simulations at each control step (in seconds).

and coordinated freeway network-control problem by employing distributed controllers. The proposed method was applied to the problem of optimal ramp metering and variable speed limits in an MPC framework. Based on the simulation results, the proposed method (DOF) achieved better performance in terms of both solution quality and computation time than those for COF. Because of the parallel nature of its solution process, the proposed algorithm can be implemented in parallel in multiple CPUs, making it potentially feasible for real-time implementation in large-size freeway networks.

For future works, we will be focusing on testing the proposed method for larger networks, including more traffic controllers, to investigate changes in the convergence process as the number of traffic controllers increases.

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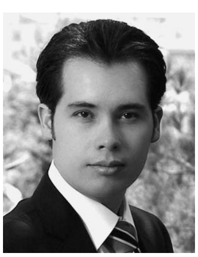

Amir Hosein Ghods (S'07) received the B.Sc. degree in electrical engineering from K. N. Toosi University of Technology, Tehran, Iran, in 2004 and the M.A.Sc. degree in electrical engineering from the University of Tehran in 2008. He is currently working toward the Ph.D. degree in transportation engineering with the University of Waterloo, Waterloo, ON, Canada.

He has provided technical service for the Road Maintenance and Transportation Organization and Tehran Traffic Control companies in Iran. His research interests lie in the area of intelligent transportation systems, traffic control, driving behavior modeling and simulation, and road safety.

Dr. Ghods is a member of the IEEE Intelligent Transportation System Society and the Institute of Transportation Engineers.

Liping Fu is a Professor with the Department of Civil and Environmental Engineering, University of Waterloo, Waterloo, ON, Canada. He has a track record of research contributions to the areas of intelligent transportation systems, traffic engineering, public transportation, winter road maintenance, and road safety, including over 80 journal and conference publications, five software packages for transportation applications, and one patent. He is the developer of a state-of-the-art paratransit scheduling system and two simulation software packages that can be used to evaluate transit performance under realistic operating conditions and management strategies. With his particular interest in integrating results of leading-edge academic research with practical applications, he continues to provide technical services for many transportation agencies, including the Ontario Ministry of Transportation, Transport Canada, the City of Edmonton, ITS Canada, and the U.S. Department of Transportation.

Dr. Fu is the Chair of Transportation Division of the Canadian Society of Civil Engineering, a member of the Transportation Research Board's Paratransit Committee, a member of the Intelligent Transportation Systems Society of Canada, and a member of Canadian Urban Transit Association.

Ashkan Rahimi-Kian (SM'08) received the B.Sc. degree in electrical engineering from the University of Tehran, Tehran, Iran, in 1992 and the M.S. and Ph.D. degrees in electrical engineering from Ohio State University, Columbus, in 1998 and 2001, respectively.

He was the Vice President of Engineering and Development with Genscape, Inc., Louisville, KY, from September 2001 to October 2002 and a Research Associate with the School of Electrical and Computer Engineering (ECE), Cornell University, Ithaca, NY, from November 2002 to December 2003. He is currently an Associate Professor of electrical engineering (Control and Intelligent Processing Center of Excellence) with the School of ECE, College of Engineering, University of Tehran. His research interests include bidding strategies in dynamic energy markets, game theory and learning, intelligent transportation systems, decision making in multiagent stochastic systems, stochastic optimal control, dynamic stock market modeling and decision making using game theory, smart grid design, operation and control, estimation theory and applications in energy and financial systems, risk modeling, and management in energy and financial systems.

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